

Effets de l'intérêt composé*

Mr. Jules Bienaymé

Soc. Philomat. Paris Extraits, Ser. 5, pp. 60–65. *L'Institut*, 286, Vol. 7, pp. 208–209.

Addition to the session of 25 May 1839

SCIENCES OF OBSERVATION: Mr. J. Bienaymé calls the attention of the Society to an effect of compound interest which touches on the conditions of existence of a crowd of establishments of which the prosperity is based on the power of the composition of the interest of money and on the aleatory chances of certain operations; namely the companies of assurances on life, the cases of retirements,¹ of annuities or of pensions.

“There is no one who does not know, he says, that the value of a sum is increased very rapidly if it is placed at interest during many years and that each year one replaces the touched interests. Reciprocally, when a sum payable in a distant epoch must be considered in its actual value, one knows that the interest, which takes then the name of discount, reduces it singularly. Thus a million francs payable in one hundred years would be worth today only very nearly 7600 francs, the interest being at 5%.

“One imagines thence that an establishment subject by the nature of its affairs to some natural gains or losses, is able to hope only the ones are compensated by the others or at least by a not very high reserve fund, if it is when the losses, arrived before the gains, are not separated for too long a time from those gains which will finish by being realized with the years. For the distant gains, although equal nominally to the losses of today, would be reduced to some sums quite inferior by the effect of compound interest. For example, for an enterprise susceptible to win or to lose annually one million, the loss of one million as of the first year would not be able to be compensated by the gain of one million in the hundredth year: since this last sum would be worth only around 7600 francs cash. If there were no closer gains, an enormous reserve fund would be necessary.

“These considerations are extremely simple: but they had not been announced, and one just sees that the consequences have some importance.

“There results from it effectively that the observations which repose on the accumulation of the interests must be in all necessity such, that the losses and the gains, plus the reserve funds, are able to be compensated in a comparatively short time.

“Now the essential condition of the compensation of which there is concern, has for a long time been established by Jakob Bernoulli, and later exposed with more clarity

*Translated by Richard J. Pulskamp, Department of Mathematics & Computer Science, Xavier University, Cincinnati, OH. June 18, 2010

¹*Translator's note:* i.e. pension funds.

by Laplace. It is necessary that the operations executed are in very great number. The two geometers cited have shown how the oscillations of the loss or of the gain, above or below of the mean result determined by the chances proper to the establishment, are not delivered at random only if the operations remain little numerous. On the contrary these oscillations are contained soon in quite narrow limits, when the operations are very multiplied. An establishment, an enterprise depending upon aleatory chances (and nearly all of them dependent), must therefore multiply its affairs much. But Laplace having not taken account of compound interest, one sees according to that just said that it is necessary to the condition of a great number to add indispensably the condition of a very short time, finally that the action of the interest not become sensible; so that the success is able to acquire that degree of probability that Bernoulli has called *moral certitude*, and on which are founded nearly all the human actions, at least the enterprise embraces a very vast sphere of nearly simultaneous operations. When it is able to join annually only a feeble number of affairs, although at length it is found by accumulating great quantities, it falls into the domain of wager, and its existence becomes very uncertain.

“There is even a remark that it is not necessary to forget: it is that the duration of an enterprise is so much less assured as the interest which forms the base of its calculations becomes more elevated. Then, effectively, the number of years during which some sums remain nearly comparable diminished considerably. If it is necessary 100 years at 5% for 1000000 fr. it is reduced to 7600 fr.; there will suffice 51 years, when the interest will be raised to 10%. So that this elevation of interest, which is able to excite to believe some cases of insurances, of pensions for widows, for the elderly, and even of other institutions worthy of the highest sollicitude, this elevation becomes immediately a source of advantages or a cause of ruin for these establishments. A continual vigilance is necessary therefore on the part of administrators, and especially of great enlightenment in the founders; because the lot of individual numbers depend on their prudence.

“Until here their attention had not been arrested on the necessity of a short time. One seemed to have believed generally that some establishments of which the duration ought to be more than secular would reunite always enough operations in order to satisfy to the one condition signaled: that of the very great number. By a natural consequence, one has multiplied without measure the assurance societies; it is what guarantees some risks of which they will never be able to gather an annual number so much is relatively low. One has of same fractioned the cases of retirements; and there exists for some administrations which do not have a pension to give each year.

“It is easy to see that these mistakes are able to involve some very great inconveniences. It is no longer there that of true wagers, it is necessary to repeat. One will sense it better still if one approaches from the action of the interest the possibility of very considerable gaps between the mean results of very numerous events of like nature; a possibility that I had had recently the occasion to indicate to the philomatic Society (V. *L'Institut*, n° 284). This possibility, who depends on the law of succession of an assembly of constant causes, renders a very great number of affairs quite more necessary than Laplace had believed it, and it constrains establishing a very strong reserve, if one has place to believe that the causes not succeed themselves at some long intervals. The isolated periods in which the compound interest obliges restraining the

compensations, are able in effect offer then some very different results. The reserve suffices for the one of these periods would become hence completely insignificant for another.

“It is therefore very important to assemble into one single establishment most possible operations. The cases of retirements especially, which are never able to attain annually but a limited number of affairs, are not susceptible to be fractioned; and it is necessary to the success as to the economy that all the retirements and the reserves which have for object to feed them, are concentrated in one institution alone. Out of a great number of cases, the chances of loss or of gain would be distributed nearly inevitably in such manner as a great part of the cases would be ruined promptly, while the other part would be enriched beyond all the possible needs; and the ruin as the richness would be able to depend very little on the management more or less good of the directors charged with these establishments. However one would attribute to them without any doubt, in the ignorance where one is of the laws which regulate the operations of this kind.

“The principles which are just exposed have daily an interesting application. I wish to speak of the position of the small commerce, what one accuses, not without reason perhaps, to show too great a roughness in the research on its benefits. One does not know that the small merchants is most subjected to the action of a higher interest; one knows equally that he is not able to multiply his affairs much. There remains to him, consequently, in order to wager with some certitude to the possible losses, even imminent, according to the most certain rules of the calculus of probabilities; there remains to him no other resource than to exaggerate the benefits. There exists, one sees, a real and mathematical motive to this exaggeration of gain, if often reproached with justice in other regards, and that the moral must proscribe when it exceeds the legitimate price of the industry. One will find probably not without utility this positive explication of a well-known economic effect, but little studied in its causes, because it seems that the cupidity of men sufficed to produce it.

“All that which precedes has been deduced from a formula of the theory of probabilities of Laplace, by introducing the effect of compound interest, of which certain financial questions required the use. I will observe (first with some surprise) that this introduction renders, to a certain degree, the illusory formula, since the great number of operations which must enter as divisor is found replaced by a number much less considerable. The divisor is no more than the number of affairs of a single year, multiplied by a quantity nearly inversely proportional to the double of the rate of interest, and approaching rapidly a limit fixed in measure as the number of years is increased. If although the probability of the gaps determined between which will be contained the final result of the operations no longer increases indefinitely with their number, as the formula of Laplace indicates in general. On the contrary, it becomes constant and even less considerable in the case of a frequent application. It is indispensable, in order that this probability is increased and approaches certitude, that the number of annual affairs become very great, since there remains a sole variable divisor.

“I have recognized later that Mr. Poisson had announced, in the *Connaissance des Temps* for the year 1827 (pag. 280 and following), the exception to which the formula of Laplace was subject. As Mr. Poisson said expressly that one is not able to take account of this exception, expecting that it is without doubt not encountered in practice,

it was clear that its habitual penetration had it only lead to remark this result bizarre in appearance. Indeed, he has well wished me to say, it is many years ago, when I will speak to him of the problem of compound interest, that his first remark had been made proper to a question on the heat contained in a closed space and reflected by the walls. The two integrations of this problem of heat, and of the probability of the mean results, when each partial result is found multiplied by a term of a series of arbitraries decreasing below each limit, being of same kind, the exception must exist at the same time in the two problems.

“The action of compound interest is thus a new example of the encounter of a real fact to which a completely analytic formula just is adapted, a formula which had appeared first only an algebraic luxury not very susceptible of application.

“I suppress here, in order to avoid the algebraic signs, the modifications that the formula of Laplace undergoes in the case of compound interest, which he had not considered.”