De la loi de multiplication et de la durée des familles.*

Jules Bienaymé

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Mr. Jules Bienaymé read a work having for title: De la loi de multiplication et de la durée des familles.

One is much occupied with the possible multiplication of the number of men; and recently diverse very curious observations have been published on the fatality which would be attached to the bodies of nobles, of bourgeoisie, to the families of illustrious men, etc.; a fatality which, says one, made vanish inevitably that which one has named *familles fermées*.[1]

These publications carry one of the members of the Society, Mr. Jules Bienaymé, inspector general of finances, to submit some results to it which he had arrived on the same subject, in researching the elements capable of making to judge on the duration of life in past centuries.

The multiplication of human kind depends especially on the magnitude of the ratio that conserve the numbers of individuals of two successive generations. If this ratio, superior to unity, expresses that the new generation surpasses in number that which it replaces, one imagines that this effect is not able to arrive unless the common fecundity of marriages exceeds the mean of the number of births necessary in order to furnish a couple who attain to the state of marriage. In other terms, the increase of the number of men and the maintenance of the races are linked to the excess of the ratio of the actual births to the marriages which produce them on the ratio of the past births to the marriages that they have produced. But at the same time in other circumstances they direct the march of the generations in the course of time; and one of the causes which influence most strongly this march is found in the combination of the values of the probabilities to have 1, 2, 3, 4, etc., or a great number of children, or to not have them at all.

In integrating by a process proper to this kind of questions the equation in differences (of the first order, but of a degree equal to the maximum of the number of children

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[1]*Translator's note*: *familles fermées*, that is *closed families*.

in a household) which results from the research of the probability of the existence of a family after a given number of generations, Mr. Bienaymé has recognized that this probability diminishes very promptly. So that under some rather favorable conditions a great number of families died out in few centuries, although the number of men is multiplied with a very great probability.

If the ratio from one generation to the other, or the mean of the number of male children who will replace the number of males of the preceding generation, were less than unity, one would imagine without difficulty that the families died out by the disappearance of the members who compose them. But the analysis shows moreover that when this mean is equal to unity, the families tend to disappear, although less rapidly. Thus, for example, if there were equal chances in order that each man had two boys or had not of them (that which would give a boy reproduces for a man come into the world, a little more with two boys or four children per marriage), of one hundred families, there would probably subsist of them only five to the 5th generation, that is, at the end of 11 to 12 centuries, by counting with Herodotus 3 generations per century. At length all families would die out, very probably at least.

It is evident from this analysis that the mean of which there comes to be question must be superior to unity, since historical times count around 200 generations. But as more of a people is erased from the globe, that number of races die out daily, and it is quite doubtful that the earth is much more populated than once, one is led to a very remarkable consequence: it is that the ratio of the generations could not have a permanent value, and that it must sometimes pass unity, sometimes fall below. Thus a population is able to subsist in the stationary state as the first authors of tables of mortality have supposed in their calculations. It is thence a proof more of the inevitable inexactitude which blemishes these calculations, and which strikes nearly all the ideas that they have given birth on the duration of life. It is equally a new example of a mean value to which great numbers of observations could not give stability. To this subject Mr. Bienaymé reports that he has exposed how Mr. Poisson had not demonstrated that which he believed to have proved on a law which would regulate *large numbers*.

The analysis would show also clearly that the mean ratio was superior to unity, the probability of destruction of families can no longer be changed into certitude with the aid of time. It does only to approach a finite limit, rather easy to calculate, and which offers this singular character to be given by that of the roots of the equation (where one makes infinity the number of generations), which do not agree with the question when the mean ratio is inferior to unity. There is therefore a sort of discontinuity which appears by the divergence of the series found for the integral of the equation in the differences.

One knows, thence, that in order to estimate that which one affirms of the extinction of the noble races, or other designated families, it would be necessary before all to know what is able to be the value of this limit during the years where they have lived. Perhaps one would find that they have left some rather numerous offshoots proportionally to their multitude.

As for the idea which appeared to have been emitted to compare the mean duration of these families with the duration of the rest of the families which form the nations, it has not practical execution. All at most would be able to compare this mean to

the duration of the extinguished nations, if it was known; because for the duration of families, of living nations, it is manifest that it goes up to the first creation, that it is indefinite, and could not furnish a subject of comparison.

Mr. Bienaymé develops diverse other considerations that the elements of the question have suggested to him, and that he supposes to publish soon in a special memoir.