AN APPLICATION OF THE CALCULATION OF THE PROBABILITY TO THE RESEARCH EXPERIMENTS OF AN APPROXIMATE VALUE OF π

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1. We begin by tracing on a plane a system of parallel lines, having between them a constant distance a, and let drop on this plane a small rod of length l (l being smaller of a). One finds then that the probability that this small rod drops so as to cut one of the parallels is expressed by

$$\frac{2l}{\pi a}.$$

If the experiment is repeated more than one hundred times, the relationship of the number of favorable cases to the total number of casts will be appreciaably equal to this fraction, whence results a relation that allows us to calculate π.

2. In 1855 M. A. Smith of Aberdeen made 3204 tests, and he deduced $\pi = 3.1553$.

A student of Prof. De Morgan found $\pi = 3.137$ after 600 tests. In 1894 Captain Fox recommenced the experiment 1120 times, with some additional precaution, and he obtained the mean value $\pi = 3.1419$.

I have resumed the experiments of these authors, and the result obtained by me forms the object of the present memoir.

3. The first thing necessary for being able to make a great number of observations, with maximum certainty and with least possible time, was to construct an apparatus able to make the small rod drop with sufficient rapidity, and to record automatically the total number of casts, as well as the number of favorable cases.

The apparatus devised by me, and of which I myself am served for the experiments of which later on, consists of three main parts:

1°. A sheet metal cylinder of thin iron, the height of 16 cm. and diameter at the base of 17 cm., open at one end and fixed through the other to an axis, put into rotation from a movement of clockwork. In the inside of this cylindrical box I put the small rod, which, by means of two stops disposed internally to the same cylinder along two diametrically opposed generators, comes through a certain length a discharge with it from the cylinder in its rotation, and therefore let to drop. But it meets the other stop here, that, in its turn, the door at the top releases it to drop, having thus an uninterrupted series of drops as long as the rotation of the cylinder endures. A counter, joined to the movement of clockwork, indicates the number of revolutions with the same cylinder, and therefore also, having two drops of the small rod for every turn, the total number of drops. In my case, the cylinder completed 12 turns to the minute, and therefore I had 24 drops of the small rod.

2°. The second part of the apparatus used by me, is that which I will call the *screen*, consisting in a rectangle of thin thread of iron, 8 cm. at base and 15 of height of which two opposite sides are joined by means of a system of thinnest iron threads, very taut, and exactly parallel between them and to the other two sides of the rectangle. This screen becomes placed horizontally in the inside of the cylindrical box of which above, and it

Date: September 14, 2009.

makes the office of the system of straight parallels traced on the paper, employed by the others, that, before me, itself is taken of the subject.

$3°$. What remains now to automatically record the number of the favorable cases. For this, the screen is capable from a pole, moving around a horizontal axis, and supplied to the other end of a writing tip. An appropriate motivating force slightly presses this tip over a strip of telegraph paper, that is made to slide under from the same movement of clockwork that puts the cylinder into motion. The tip therefore comes to trace on this strip a continuous line. But we suppose now that the small rod, of which above, comes, in the dropping, to hit against one of the threads of the screen: this that, as I have said, is most mobile around to a horizontal axis, lowers, while the writing tip is raised from the opposite part, and the line remains interrupted from tracing. But quickly the motivating force leads the tip back into contact with the paper, and in this way every favorable case is marked by an interruption in the line.

Thus being the things, loaded one time the movement of clockwork, we can abandon the apparatus to itself, remaining to us nothing, at the end of the experiment, than to count the interruptions on the strip, and to replace this value, at the same time with that one I give to us from the counter of the turns multiplied for 2, in the formula

$$\frac{f}{t} = \frac{2l}{\pi a},$$

where
$f =$ number of favorable cases,
$t =$ number total of drops,
$l =$ length of the small rod,
$a =$ height of the strip limited from two spins parallels of the screen,

from which

$$\pi = \frac{2lt}{af}.$$

4. And here the result from me obtained with this means.

$l = 2.5, a = 2.6$

drops		
Total No.	Favorable No.	Value found for π
100	60	3,205
500	276	3,483

From before I had joined between they smaller sides of the screen, in a way that the system of the parallel threads turned out parallel to the generators of the cylinder: under these conditions the approximation has been very poor, as can be seen here from the united table.

5. I have had instead better approximation with disposing the screen cross-sectionally, it is worth saying to join between them larger sides of the rectangle. Here the experiments go uniform in two series, since, while I have maintained the length of the small rod always constant, have made instead the height of the strip comprised between the parallels to vary: and here the result obtained:

I$^{\text{st}}$ Series $l = 2.5, a = 2.6$				II$^{\text{st}}$ Series $l = 2.5, a = 3$		
drops				drops		
Total No.	Favorable No.	Value found for π		Total No.	Favorable No.	Value found for π
100	62	3.101		100	53	3.144
200	122	3.152		200	107	3.115
1000	611	3.147		2000	1060	3.1446
2000	1229	3.126		3000	1591	3.142
3000	1840	3.135		3408	1808	3.1415929
4000	2448	3,142		4000	2122	3.1416

Reassuming, we have in the event that the obtained values more near to the value of π are

drop No.	Value Found No.
100	3.144
1000	3.147
2000	3.1446
3000	3.142
3408	3.1415929
4000	3.1416

that is, of 3408 drops, with an error smaller than $\frac{1}{5{,}000{,}000}$.