Mémoire sur l'application de la nouvelle formule d'interpolation à la détermination des orbites que décrivent les corps célestes, et sur l'introduction directe des longitudes et des latitudes observées dans les formules astronomiques*

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In the last session, I am arrived to this remarkable result, that the radius τ drawn from a comet or from a planet to the earth, at a given epoch, is able to be furnished by a very simple equation of the first degree, of which the coefficients, usually, are able to be determined, at least approximately, by aid of four observations made at some instants neighboring the epoch of which there is concern. If one names ρ the projection of the radius τ onto the plane of the ecliptic, and r the distance from the sun to the comet, the three equations of movement will furnish, beyond the known equation of the seventh degree, the values of $D_t\rho$ and of $D_t^2\rho$ expressed as functions of ρ. By differentiating $D_t\rho$, one will obtain a second value of $D_t^2\rho$, and, by equating this second value to the first, next eliminating $D_t\rho$, one will form the equation mentioned above. If, as Mr. Binet indicates, one completed the equations of movement introducing in them the terms which depend on the action exercised by the other planets on the comet, the found equation in ρ will no longer be of the first degree; but one would be able, from this equation joined to that which Mr. Binet has given, deduce an equation of the first degree, by making the radical disappear, and recurring next to the method of the greatest common divisor. The radius ρ being known, so that its derivatives of the first and of the second order, the coordinates of the comet with their derivatives relative to time, and, hence, all the elements of the orbit are thus known. Besides, one is able to arrive in diverse ways to the equation of the first degree, even when one considers three bodies only. One has regard how difficult the determination of the geocentric longitudes and latitudes and of their [957] derivatives corresponding to a given epoch. But this difficulty disappeared, when one applied to this research the formula of interpolation that I have found in 1837. As I will demonstrate it in a forthcoming Memoir, the operation is divided then into two others, of which the one

*Translated by Richard J. Pulskamp, Department of Mathematics & Computer Science, Xavier University, Cincinnati, OH. July 8, 2010 determines some numbers which depend uniquely on the epochs of the observations, while the other employs only the longitudes and latitudes deduced from these same observations.

There remains for me to make yet an essential remark. The formula which I have given in the last session assumes the geocentric longitudes and latitudes each corrected by the quantity which represents aberration. It seems, at the very first, that these corrections require a preliminary approximate calculation. But one is able to render my equation of the first degree, or even all the astronomical formulas, independent of the correction of which there is concern, and introduce into these formulas, instead of the corrected geocentric longitudes and latitudes, the apparent geocentric longitudes and latitudes, directly drawn from the observations. This which will not be able to be lacking in interest the astronomers, it is the conclusion to which I arrive; namely, that, in this case again, the equation is obtained, with respect to ρ, of the first degree.

ANALYSIS

We admit the same notations as in the preceding Memoir. After having determined ρ and $D_t\rho$ by aid of the equations

$$(1) \qquad C\rho = B - A^2 - D_tA - \frac{1}{R^3}, \qquad (2) \qquad D_t\rho = A\rho,$$

one will determine x, y, z by aid of the following

$$(3) \qquad x = \mathrm{x} + \rho\cos\alpha, \quad y = \mathrm{y} + \rho\sin\alpha, \quad z = \Theta\rho.$$

By differentiating these last, one will obtain the values of D_tx, D_ty, D_tz. If besides one names $2S$ the area described, during the unit of time, by the radius vector drawn from the sun to the star that one considers, and $2U$, $2V$, $2W$ the algebraic projections of this area onto the axes, one will have

$$(4) \qquad U = yD_tz - zD_ty, \quad V = zD_tx - xD_tz, \quad W = xD_ty - yD_tx,$$

$$(5) \qquad S = \sqrt{U^2 + V^2 + W^2};$$

and, as the quantities

$$U, V, W$$

[958] will be respectively proportionals to the cosines of the angles formed by the perpendicular to the plane of the orbit with the axes, it is clear that the only knowledge of these quantities or rather of their ratios will give immediately the position of the plane of the orbit. We add that the distance r from the star to the sun, and its derivative D_tr, will be determined by the equation

$$(6) \qquad \frac{1}{r^3} = \frac{1}{R^3} + C\rho$$

and by its differential. Finally, if one names ω the speed of the star, a the semimajor axis of the orbit, and ε the eccentricity, one will have

$$(7) \qquad \omega^2 = (D_tx)^2 + (D_ty)^2 + (D_tz)^2,$$

(8) $$\frac{1}{a} = \frac{2}{r} - \omega^2,$$

(9) $$a(1-\varepsilon^2) = 2r - \frac{r^2}{a} - r^2(D_t r)^2.$$

We say now some words on the correction that the aberration requires in the determination of the radius ρ.

One demonstrates easily the following two propositions:

1st *Theorem*. The radius vector drawn at the end of time t from the earth to the apparent place of the star that one considers, is sensibly parallel to the radius vector which joined the earth to the true place of the star, at the end of time $t - \Delta t$, Δt being the time that the light uses in order to come from the star to the earth.

2nd *Theorem*. The radius vector drawn from the earth to the true position of the star, at the end of time t, is sensibly parallel to the radius vector which will join the earth to the apparent position of the star, at the end of time $t + \Delta t$.

This put, let

(10) $$\rho = K$$

be the value of ρ furnished by equation (1). Let besides H be the part of $D_t K$ which one obtains by considering, in K, α and θ alone as functions of t, that is to say by rejecting only the terms which the differentiation of R ϖ, and $D_t\varpi$ produces. When one will assign to the quantities α, θ and to their derivatives, the values that one deduces from the observations, one will have sensibly, by virtue of the second theorem,

(11) $$\rho = K + H\Delta t.$$

[959] If besides one names δ the speed of light, and τ the distance from the earth to the star that one considers, one will find

(12) $$\delta\Delta t = \tau, \quad \rho = \tau\cos\theta;$$

consequently,

(13) $$\Delta t = \frac{\tau}{\delta} = \frac{\rho}{\delta\cos\theta}.$$

Therefore formula (11) will give

$$\rho = K + \frac{\rho}{\delta\cos\theta}H;$$

and one will conclude from it

$$\rho = \frac{K}{\left(1 - \frac{H}{\delta\cos\theta}\right)},$$

or, very nearly

$$\rho = K\left(1 + \frac{H}{\delta\cos\theta}\right).$$