

# Substitutions\*

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*Encyclopédie Méthodique*  
Mathématiques. Tome I, 1<sup>ère</sup> Partie, 1783

SUBSTITUTIONS<sup>1</sup> (*Calculus of probabilities*) It appears difficult to refuse to the legislative power the right to change the laws of successions; otherwise it would be necessary to sustain, 1<sup>o</sup> that these laws are absolutely arbitrary, since it would be absurd to refuse to the legislative power the right to destroy the laws which are contrary to reason. 2<sup>o</sup> That these laws are useful, whatever they are able to be, provided they are permanent; since by supposing the laws arbitrary, that is, having no other motive than general utility, & not justice, it would be absurd to refuse to the legislative power the right to destroy the injurious laws. 3<sup>o</sup> Because it would be necessary to regard the right which results from these laws, as dependent on the common will of men who exist in the moment where they have been made, & to suppose to them a superior authority to that of the common will of the men who existed: this which would be absurd, at least by supposing these laws the first which had ever been made on the successions.

Also the Jurisconsultants who have contested this right, have rather refused it to such legislative & particular bodies, to which they supposed that nature had entrusted only a limited authority, & who were compelled to conserve the anterior laws to his institution. It is true that a constitution where there would be no way to reform the bad laws, would cease by that alone to be a legitimate constitution; but the Jurisconsultants have not always had enough light in order to sense the reasons; & besides, attached by state to the tribunals, the spirit & the interest of bodies or of profession, should render them favorable to the perpetuity of the laws.

But, supposing even that there exists an authority which is able to change the civil laws, & in particular, those laws on successions; all the time that there will result from this change injury from a right acquired by the ancient laws, justice requires that this right is evaluated & replaced by a value which is equal to it.

It is necessary therefore to examine in what case there can exist rights of this kind, & next the calculation of the probabilities will teach the ways to evaluate them.

We will suppose that there is only one to be existing who is able to have some rights, & that there is of rights only those which are assured by the law; in a manner that any particular will is not able to deprive it; this which restrains the rights, & consequently the compensations to those which are able to give the *substitutions* & the eventual

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<sup>1</sup>A *substitution* is an entailed estate. That is, the estate is limited to a certain line of heirs or successors in such a way that it can never be legally transferred.

donations. We suppose in fact that there is question of birthright, & that the law, by abrogating this right, substituted an equal share, it is clear that, in the case of the custom or of the ancient law, the father being able to sell his wealth, & to share the product in silver or in notes to his children, & being able, according to the new law, to dispose of this same produce in favor of the eldest alone; the law produces no change except in a manner subordinated to the will of the father, & change consequently nothing to the acquired rights of the children.

We will speak therefore here only of the single case where the law would destroy some established *substitutions*, because the case of the eventual donations would be calculated easily, according to the same principles.

It would be necessary first to limit the right of *substitution* to the individuals called actually *living*, in a manner that the wealth becomes free among the hands of the last from among them, or to the dead, if, according to the rule of *substitutions*, it has right to pass to some persons born after the promulgation of the law.

2° To establish that the actual possessor would acquire the liberty to dispose, by reimbursing to the called to the *substitutions*, the value of their rights. Let therefore  $P$  be the possessor,  $S'$ ,  $S''$ ,  $S'''$ ,  $S''''$  ... the called, by supposing that  $S'$  is the first,  $S''$  the second,  $S'''$  the third; let  $V$  be the value of the *substitution*, &  $r$  a quantity such that the actual value of  $V$ , to receive at the end of a year is  $Vr$ , it is clear 1° that if  $p'$ ,  $p''$ ,  $p'''$ ,  $p^n$  express the probability that  $S'$  will survive to  $P$  at the end of 1 year, of 2, of 5, &c. the actual value of  $V$  for  $S'$  will be

$$V.(p'r + p''r^2 + p'''r^3 \dots + p^n.r^n)$$

a quantity that it will be necessary to divide by  $r^{\frac{1}{2}}$  in order to have the mean value which results from the hypothesis that the period of life will arrive equally in all the moments of each year.

If  $S'$  is not called necessarily (this which takes place if  $S'$  is not the eldest son of  $P$ , & if  $P$  consequently is able to have a child who is called after  $S'$ ),  $p'$ ,  $p''$ , ...  $p^n$  express then the probability that  $S'$  will survive into the first year, into the second & into the third to the children that  $P$  would be able to have. But the wealth becomes free on the head of  $P'$  only in the case where he survives to  $S''$ , to  $S'''$ , to  $S^{iv}$ ; therefore calling  $p'$ ,  $p''$ ,  $p'''$ ,  $p^n$ , the probabilities that he will survive them into the first, into the second, into the third, into the  $n^{\text{th}}$  year, we will have, for the value of the right of  $S'$

$$\begin{aligned} & \frac{V}{r^{\frac{1}{2}}} [p'(p'r + p''r^2 \dots + p^n.r^n + \overline{1-r}(1-p') \\ & + r.1 - p' - p'' + r^2.1 - p' - p'' - p''' \dots + \frac{n-1}{1-p' \dots})) \\ & + p''(\overline{p' + p''r^2 + p'''r^3 \dots + p^n.r^n} + \overline{1-r}(r^2.1 - p' - p'' \\ & + r^2.1 - p' - p'' - p''' \dots + p^{n-1}.1 - p' \dots)) \\ & + p'''(\overline{p' + p''p''r^3 + p^{iv}.r^4 \dots + p^n.r^2 \dots} \\ & + \overline{1-r}(r^2.1 - p' - p'' - p''' \dots + p \frac{n-1}{1-p' \dots}))] \end{aligned}$$

a formula which is easy to continue, or by ordering with respect to  $r$ .

$$\begin{aligned} & \frac{V}{r^{\frac{1}{2}}} \cdot [p' \cdot p'_i + (p' \cdot p'' + \overline{p'' \cdot p'_i + p''})r^2 \dots \\ & \quad + (p' \cdot p''' + p'' \cdot p''' + p''' \overline{p'_i + p'' + p''})r^3 \\ & \quad + (\overline{p' + p'' + p''} p_i^{iv} + p^{iv} \overline{p'_i + p'' + p'' + p^{iv}})r^4 \\ & \quad + \overline{1 - r} (p' \cdot \overline{1 - p'_i + p' + p''} \cdot \overline{1 - p'_i - p''} r + \\ & \quad \overline{p' + p'' + p''} \overline{1 - p'_i - p'' - p''} r^2 + \dots \\ & \quad p' + p'' + p''' + p^{iv} \cdot (1 - p'_i - p'' - p'' - p_i^{iv} r^3 \dots).] \end{aligned}$$

And this function expresses that which  $P$  must give to  $S'$ . In order to have that which  $P$  must give to  $S''$ , one will form a similar function by supposing only that the  $p'$ ,  $p''$ ,  $p'''$ , represent the probability that  $S''$  will survive to  $P$  &  $S'$ , as to those who, being born to them, would have right to the substitution in the 1<sup>st</sup>, the 2<sup>nd</sup>, the 3<sup>rd</sup> year, to  $p'_i$ ,  $p''_i$ ,  $p'''_i$  ... the probabilities that  $S''$  will survive to  $S'''$ ,  $S^{iv}$  ... in the 1<sup>st</sup>, the 2<sup>nd</sup>, the 3<sup>rd</sup> year, & thus consecutively for the others. All the quantities above are given by the tables of mortality, excepting that which will depend on the probability that a man marries or not marries, of a given age, will not have children, or that the children that he will have will be born at a given age, a quantity that one would find easily according to the same tables of mortality, & the tables for the fecundity of marriages, & the ratio of the celibate or married men; but until here these last two kinds of tables are very defective, because one has not marked the age of the husband of whom one seeks to determine the mean fecundity, & that one has set only in bulk & not for each age, the ratio of the celibate to the married people.

One would be able perhaps to fear that the possessor had never tried to pay the compensation; & that thus a law, such as we indicate, will not accelerate the destruction of the *substitutions*. But 1<sup>o</sup> it is easy to see, by following the method of evaluation that we have exposed, that there exists a great number of circumstances where this compensation would be very feeble & below the wrong that the owner of the attached trouble receives in the *substitutions*; that besides a father would be only held to compensate the collateral, & to assure to his children born & called to the *substitutions*, the value of their rights according to him. 2<sup>o</sup> That the creditors could make sell the substituted wealth, by being charged from the compensation.