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GAGEURE

GAGEURE, (*Analyzes of chance*.) is the same thing as *pari*, which is more used in this encounter. *See* **Pari**, **Jeu**, & **Gageure** (*Jurisprud.*)

This article provides us an opportunity that we sought to insert here some very good objections which were made to us on what we have said *in the word* **Croix ou Pile**, on the matter of calculating the advantage in this so common game. We pray the reader to please first reread well the beginning of this article **Croix ou Pile**. Here are now the objections that we just announced. They are by Mr. Necker junior, citizen of Geneva, professor of Mathematics in that city, correspondent of the Royal Academy of Sciences of Paris, & author of the *article* **Frottement**; we have extracted them from one of his letters.

> One asks the probability that there is produced heads in two throws. You say that there are only three possible events, 1°. heads first, 2°. tails & heads, 3°. tails & tails; & as two of these events are favorable & one harmful, you conclude that the probability of producing heads in two throws, is of two against one, This conclusion supposes two things; 1°. that this enumeration of all possible events is complete; 2°. that they are all three equally possible, *aequè proclives*, as says Bernoulli. I agree with you of the truth of the first point; but we differ on the second point. I believe that the probability of producing heads first is double of the one of producing tails & heads or tails & tails. The direct proof that I believe to have of it, is the one here. It is as easy to produce heads first as tails first; but it is much more probable that one will produce tails first, than tails & heads: for in order to produce tails & heads, it is necessary not only to produce tails first, but after having brought tails, it is necessary next to produce heads; a second event as difficult as the first. If it was so easy to produce in two throws tails & tails as tails in one throw, it would be by the same reason again by the same facility to produce tails, tails, & tails in three throws, & in general to produce n throws; however who is it who doesn't find it incomparably more probable to produce tails in one throw, than to produce tails one hundred times in succession? Here is another way to consider the thing. Either I will produce heads on the first throw, or I will produce tails. If I produce heads, I win all the stake of the other; if I produce tails, I neither lose nor gain, because afterwards at the second cast I have an expectation equal to his. Therefore, since

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I have equal chance to have his stake or to have nothing, it is as if he redeemed all his risk, in giving to me half of his stake. Now the half of his stake that he gives me, with mine that I recover, makes $3/4$ of it in total, & the other half of his stake that he keeps makes the other quarter of the total: therefore I have three shares, & he one; my probability of succeeding was therefore of 3 against 1. But here is something more decisive. It would follow by your way, Sir, to compute the probabilities, that one would not be able in any number of throws to wager with parity to produce the face A of one die with three faces A, B, C; because you will find always of $2^n - 1$ against 2^n, n being the number of throws in which one attempts to produce the face A. Here are in fact all the possible cases in four throws, for example:

A	B, B, B, A	B, B, B, B	$C, B, B, B.$
B, A	B, B, C, A	B, B, B, C	$C, B, C, B.$
C, A	B, C, B, A	B, B, C, B	$C, B, C, C.$
B, B, A	B, C, C, A	B, C, B, B	$C, B, B, C.$
B, C, A	C, B, B, A	B, B, C, C	$C, C, C, C.$
C, B, A	C, B, C, A	B, C, B, C	$C, C, C, B.$
C, C, A	C, C, B, A	B, C, C, B	$C, C, B, C.$
	C, C, C, A	B, C, C, C	$C, C, B, B.$

It is easy to see that there are here 15 favorable cases & 16 unfavorable; in a way that there is $2^4 - 1$ against 2^4, that one will produce the face A. It seems to me therefore certain that the case A is not able to be regarded as being more probable than the case B, C, B, B, &c.

These objections, especially the last, deserves without doubt more attention. However it seems to me always difficult to explain well why & how the advantage could be triple, when there are only two favorable throws; & one will agree at least that the ordinary method by which one estimates the probabilities in these kinds of games, is very faulty, when one would claim that the result of this method would be correct; it is this that we will examine in greater depth in the *articles* **Jeu**, **Pari**, **Probabilité**, &c. (*M. d'Alembert*)