III. ON THE TABLES OF MORTALITY

JEAN D'ALEMBERT
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Some Mathematicians have asked me the proof of that which I have advanced in the preceding Volume (twenty-seventh Memoir, § II, art. 37[1]) on the subject of the different results on the probability of life, drawn from the Tables of mortality of Sweden. These tables of mortality are found in the Supplement of the Work of Mr. Deparcieux, printed in 1760, page 33. We see first easily that the half of the years which remain to elapse up to one hundred years, by commencing with 0, that is at the moment of birth, is represented by the second vertical column of the following table; and the mean life for each age, which indicates, following the greater part of the Authors, the probability of the duration of life, is expressed by the numbers of the third column. In order to have the numbers of the fourth column, which indicate, according to Mr. de Buffon, this probability, it is necessary to consider that if at 0 years there exist 10000 persons who are born, there will exist no more

[1]*Opuscules* IV, Twenty-seventh Memoir, "Sur les Calculs relatifs à l'Inoculation," pp. 340–341 and figure. 37. On the occasion of these reflections, I will make you part by finishing a remark on the curve of mortality. By examining the tables of mortality given by Mr. Deparcieux according to the Registers of Sweden, we see easily that the mean life (which is nothing other than the probability of the duration of life, taken in the ordinary manner) is always smaller than half of the years which remain to elapse up to 100 years, a term which we can suppose longer than human life; whence it follows evidently that the curve of mortality is neither a straight line, nor a curve entirely concave towards the x axis, that which it was easy to presume besides; but we can indicate moreover that the duration of life, taken in the manner of Mr. de Buffon, (see above pages 86 & 92) [i.e. Memoirs 23.VI §18 and 23.VII] is nearly always greater by this table than the mean life; whence it is easy to conclude that the curve of mortality is not entirely convex toward the x axis, indeed, if it were entirely convex, we would have by making (Fig. 20) $AC = \frac{AB}{2}$, BO (which expresses the duration of life according to Mr. de Buffon) $= CE - DE = \frac{DF}{2} - DE$; and the mean life would be $= \frac{ABFDA}{AB} = \frac{AEFD - ADFEA}{AB}$; now supposing ADF entirely convex, $\frac{ADFEA}{AB}$ is evidently $< DE$; therefore BO would be $< \frac{ABFDA}{AB}$.
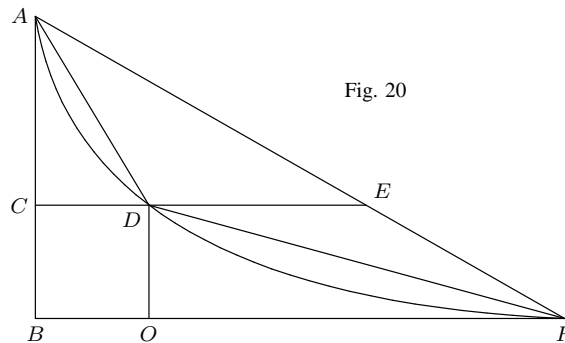

Fig. 20

according to the tables of Sweden than half or 5000 between 7 and 8 years; which to a year, there exists no more that 6939 persons, of whom the half is 3469, & that there will be no more than this half to exist between 38 & 39 years; whence it follows, by subtracting one year, that 37 to 38 will be the probability of life, according to Mr. de Buffon, for the persons aged one year; we will find likewise, by the tables, that at the age of two years there remain only 6245, of whom there exists no more that the half 3122 at the age around 44 years; whence it follows, by subtracting 2 years, that the probability of life will be 42 years, according to Mr. de Buffon, for the persons aged 2 years; likewise at 3 years there will remain only 5757 persons, of whom there will exist only the half 2878 at the age around 51 years; whence it follows that $51 - 3$ or 48 will be the probability of life, according to Mr. de Buffon, for the persons aged 3 years. By this means, we will form the fourth column of the following table. Now it is easy to see that the numbers of the second column are greater than those of the third, & that those of the third are also smaller than those of the fourth, in the years of life which extend to 40 years, excepting in the first year. We find a result nearly similar for the table of order of mortality according to the researches of Mr. le Curé de ***, which Mr. Deparcieux has joined to the table of Sweden.

Ages		Years	Months	Years	
0	50	27		7	
1	$49\frac{1}{2}$	37	1	39	3061
2	49	40	1	42	694
3	$48\frac{1}{2}$	42	5	48	408
4	48	44	0	49	335
5	$47\frac{1}{2}$	44	11	51	226
6	47	45	3	50	438
10	45	44	11	46	224
15	$42\frac{1}{2}$	42	0	42	187
20	40	38	8	40	218
25	$37\frac{1}{2}$	35	0	36	223
30	35	32	0	33	261
35	$32\frac{1}{2}$	29	2	29	228
40	30	26	0	25	279
45	$27\frac{1}{2}$	23	2	21	246
50	25	20	0	18	287
55	$22\frac{1}{2}$	17	0	$14\frac{1}{2}$	299
60	20	14	0	$11\frac{1}{2}$	380
65	$17\frac{1}{2}$	11	3	9	415
70	15	9	0	7	491
75	$12\frac{1}{2}$	7	3	6	399
80	10	6		5	299
85	$7\frac{1}{2}$	5			191
90	5	5			70
95	$2\frac{1}{2}$				29
100	0				

We can further confirm this remark by the last column of the table, which indicates the number of deaths of each year, namely 3061 in the interval of the birth to one year, 694 in the interval from one to two years, & thus in sequence. It is easy to see that this number, instead of being decreasing as it must be from year to year, if the curve of mortality were entirely convex towards its axis, increases on the contrary from 15 years to 30, & from 45

to 70; whence there results evidently that the curve of mortality is neither entirely convex nor entirely concave towards its axis, but that it has some points of inflection & serpentine shape.

There could be besides that the other tables of mortality gave some different results; I speak here only of that which has been prepared in Sweden, & which seems to have been made with care; some other tables of mortality give some very different results for the mean life; and it is in them, even among those that Mr. Deparcieux reports, which give up to 12 years difference between the results of mean lives. See table XIII of his Work. We see also in this same table, that the one of the Registers of London, the half of the infants born together are dead at the beginning of the third year; according to the tables of Mr. Halley (by supposing 1300 infants born at once) at the beginning of the eleventh year; & according to the tables of Holland, at the beginning of the thirty-first year; a considerable difference, & which proves, either how the tables of mortality differ according to the different countries, or perhaps how these tables are imperfect, & have need of being prepared with more care & precision than they have been until now.