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PHYSICAL AND ASTRONOMICAL RESEARCHES

on the problem proposed for the second time by the Academie Royale des Sciences de Paris.

What is the physical cause of the inclination of the Planes of the Orbits of the Planets with respect to the plane of the Equator of the revolution of the Sun around its axis; And whence comes that the inclinations of these Orbits are different among themselves.

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Documents which have won the prizes of the Académie Royale des Sciences de Paris 1734 (1735), p. 93–122
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Virtutum pretium in ipsis est, & recté facti merces est fecisse.

PREFACE.

I have made this translation at the request of some of my friends in Paris, to whom I owe all kinds of respect and gratitude. Those who will wish to give themselves to the pain of confronting it with the original Latin, will see that if this is not a word for word translation, at least I have guarded the sense of each sentence; but I have made some small additions or clarifications, of which I have been able to dispense with before I had known that I may have readers other than the Judges. These additions are distinguished in the body of the document by two parentheses of this form [. . .] which contain them.

I pray here the reader, at no point finds wrong the style which I have affected in speaking of my father; I am availing myself of it in order to conceal myself further to the Academicians.

§ I. The Problem which the illustrious Academy proposes, has two parts; the one regards the inclination, or the non-coincidence of the celestial Orbits with the solar Equator; the other has for object the diversity of these inclinations. We will consider both at the same time, our system does not permit that we separate them.

§ II. We see in the same manner, in which the Academy has stated its Problem, that it presupposes having a liaison between the Orbits of the Planets & the Equator of the

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Sun, which tends to put them in a common plane, & which without a particular reason the planetary Orbits will be entirely in the same plane with the solar Equator.

This has seemed to me likewise always quite probable; because could we, in order to not allege other reasons, attribute to a pure chance the slight inclination of all these Orbits to the plane of the solar Equator? Or if this might seem yet doubtful [seeing the slight precision & certitude in the position of the solar equator] at least we will not be able to deny that the planetary Orbits tend towards a common plane, because without this it would have been morally impossible, that the Orbits were contained within the limits as compact as there are. This being, it is quite probable that this plane of common tendency is the same as the one of the solar Equator, this one being the one alone in which we can find some reason capable of producing such a Phenomenon.

This put, the question is to find a physical reason, which makes the celestial Orbits bend & approach towards the Equator of the Sun, & to determine why these Orbits are not entirely, neither in the plane of the said Equator, nor in a common plane.

§ III. Before attempting these two points, it will not be irrelevant to examine more particularly what we have posed as fact; namely, *that the celestial Orbits approach much too nearly in order to not set apart some common plane situated in the middle of them, & that it is only by a particular circumstance, that the same Orbits are not entirely united in one same plane*. Without this examination, we could attribute to a chance the Phenomenon, which forms the subject of our question, & to regard all our reasoning as superfluous, or perhaps even chimera.

Here is how I myself will take it: I will seek of all the planetary Orbits, the two which cut themselves with the greatest angle, after which I will calculate what probability there is, that all the other Orbits are contained by chance within the limits of these two Orbits. We will see thence that this probability is so small, that it must to be received as a moral impossibility.

§ IV. After having compared each Orbit with each, & calculated the angle, with which they intersect themselves, I have found the Orbit of Mercury & that of the Earth or the ecliptic to cut themselves with the greatest angle; for their planes make an angle of $6^\circ\ 54'$; & while the Orbit of Saturn makes, with that of Mercury, only an angle of $6^\circ\ 24'$; & the Orbit of Jupiter, again with that of Mercury, an angle of $6^\circ\ 8'$. All the other Orbits, in whatever manner that we combine them, cut themselves with some much smaller angles. I speak here of the Orbits of the principal Planets.

[It is easy to see that we can find the said intersections by simple Trigonometry; because as we know the nodes of the Orbits, as well as their inclinations with the ecliptic, we will have in a spherical triangle for given base the distance of the nodes, & the two angles around the base will be known by the angles of inclination of the Orbits with the ecliptic. Thence we will find the angle opposed to the base which makes the angle of intersection of the two Orbits; thus, for example, we find the angle, with which the Orbits of Saturn & of Mercury cut themselves in the year 1700, by considering that, following Kepler, we have then the ascendant node of Saturn in the $22^\circ\ 49'$ of *Cancer*, & that of Mercury in the $14^\circ\ 47'$ of *Taurus*; the distance of the nodes is thus here $68^\circ\ 2'$, which makes the base of the triangle. And, following the same Author, the Orbit of Saturn cuts the ecliptic with an angle of $2^\circ\ 32'$, and that of Mercury with an angle of $6^\circ\ 54'$. We have therefore the angles around the base of $2^\circ\ 32'$ & $173^\circ\ 6'$; & seeking thence the angle opposed to the base, we find it of $6^\circ\ 24'$; as we have noted. Besides we see well that the nodes being differently mobile, the angles of intersection of the Orbits must be variables; but that is here of no importance.]

I conceive therefore all the spherical surface girded by a zone, or kind of Zodiac, of width $6^\circ\ 54'$ (because such is the greatest inclination of the Orbit of Mercury with the ecliptic.) This zone will contain very nearly the seventeenth part of the spherical surface. If we consider therefore the planetary Orbits as placed by a pure chance, the question will be to determine what degree of probability there is in order that all the Orbits fall into one zone given in position, making the seventeenth part of all the spherical surface. But the position itself of the zone is determined by one of the Orbits, whatever it be, because they scarcely differ among themselves; which makes that there are no more than five Orbits which enter by line of computation; this put, we will find by the ordinary rules, the number of case, which make the five Orbits fall within the said zone, to the number of contrary cases, as 1 to $17^5 - 1$; that is to say, as 1 to 1419856.

[I do not give by this method all the geometric precision, which the Reader will not have failed to note; but I am content in it, because the question here is to have only some general idea of the thing. A number considerably greater or smaller, would not make us consider otherwise the point of the question. We see nevertheless rather that our proportion can not be far removed from the truth. But, will you demand of me, what is therefore the truth? I respond to this demand, that we would not know how to determine it because of the movement of the nodes which change at each moment the limits of the Orbits; I have therefore simply considered one zone, outside of which any point of the Orbits, although varying in position, never follows, & I have compared this zone with the surface of the sphere, of which it makes nearly the seventeenth part, sometimes more, sometimes less, because of the variability of the limits. In this zone there is no point, which is not subject to be touched by one of the Orbits; & beyond the same zone, there is no point which can ever be; whence we see rather the foundation of my solution. If all the nodes were constantly in one same common point, it would be necessary to have regard for the greatest angle of intersection of 2 Orbits which we have seen to be of $6^\circ\ 54'$; & as this angle would have been able to go to 90°, if chance would have formed it, it would be necessary to compare these two angles, & to say that the first makes approximately the thirteenth part of the second; whence we would deduce the degree of probability (in order that none of the Orbits make with one other Orbit an angle greater than $6^\circ\ 54'$) equal to

$$1 : (13^5 - 1),$$

which gives a proportion approximately four times greater, than in the first solution, namely, that of 1 to 371292. Finally, the better method of calculating the degree of probability, would be to consider the plane in the middle of the Orbits (which, according to all appearances, is the same plane of the solar Equator) with which each Orbit, although moving, makes without doubt a constant angle, or nearly constant. If this plane were given in position, it would be necessary to calculate what Orbit makes the greatest angle with this plane, and what is the magnitude of that angle; & as in the hypothesis of the Orbits at random placed this angle would be able to rise to 90°, we would be again having to consider the ratio of the said angle with the one of 90°, & put this ratio to be of 1 to m, the degree of probability sought, would be now as $1 : m^6 - 1$. I put here the exponent 6 in place of 5, as I have put in the two preceding examples, because the fixed term is not here one of the Orbits, but the solar Equator. This method seems to me the most correct of all, if the determination of the solar Equator was a little more certain; following what M. Cassini reports in the Mémoires de l'Académie Royale des Sciences de Paris in the year 1701[1], it is the Orbit of the Earth which makes the greatest angle with the solar Equator, & this

[1]J. Cassini, *Des taches observées dans le Soleil* : Mém. Paris 1701, p.264.

angle must be $7^\circ\ 30'$, which would give

$$m = 12, \quad \& \quad m^6 - 1 = 2985983.$$

If therefore all the Orbits were placed at random with respect to the solar Equator, the odds would be 2985983 against 1, that they would not be all so close. All these methods, although quite different, do not give some extremely unequal numbers. However I will cling to the number given in the first place, & make this addition only in the design to show to the Reader what foundation we can make there.]

§ V. Someone will perhaps find fault with this method; I myself had made first another; however all things considered, I have preferred to it that which I have exposed in the first place. I will not stop myself however to confirm it, in order that it not avert me further from our principle subject.

However, in order to better sense the ridiculousness that there would be to attribute to a pure chance the compact position of the Orbits, we will compare the question of six Orbits with that of a simple intersection. I say therefore that this position of the Orbits is less probable, than would be that of two Orbits which must cut themselves with an angle smaller, than of a fourth of a second [for since the angle of 90° is to the angle of $15''''$, as 1296000 to 1, there is here only 1295999 cases against 1, instead that there we have found it to have 1419856 to 1:] now if for example Nature had given to the Ecliptic only an angle of $15''''$ inclination with respect to the Equator of the Earth, supposing that the skill of men had been able to arrive to measure of such angles, whatever one would he be able to believe that this itself was made by pure chance, without there being the least liaison between the Ecliptic & the said Equator? But if we again pay attention to the Satellites of Jupiter & of Saturn, which, like the principal Planets, make their course almost in a common plane (excepting the last Satellite of Saturn, which for a particular reason, which our theory even will specify, has not entirely this law) there will no longer be able to remain the least scruple on this matter; & who is not in this sentiment, must reject all the truths, which we know by induction. We return to our principal subject.

§ VI. We have said, that there is a plane which must have some relationship with the Orbits of the Planets, in which these Orbits try to reunite themselves; that this plane is situated in the middle of the Orbits, & finally that it is, according to all appearances, the same as the one of the solar Equator, as much because the plane of this Equator traverses effectively the middle of the Orbits, as far as we can judge it by the observations made of the sunspots, as because it is the only plane which can furnish a physical reason at this point. After what we have added, that it must have a particular circumstance, with respect to which the planetary Orbits can not be entirely united in the plane of the solar Equator, or in a common plane. It is in these two points which consists principally the proposed question. I sense therefore, that in order to satisfy the demand of the Academy, I must first demonstrate, what can have drawn the planetary Orbits so near to the solar Equator; & in second place, why these Orbits are not entirely united with the same Equator.

§ VII.–§XX. are devoted to Bernoulli's hypothesis of the solar atmosphere as cause.

§ XXI. After having advanced several reasons in order to prove that the Planets tend around the equator of the Sun, & that they near it the more and more; it will be good to examine here, by Astronomical observations, what is the inclination of the Orbits with respect to the said equator; in order to know it, it is necessary to know the position of the nodes, or intersections of the planetary orbits with the ecliptic, & finally the position of the solar equator with respect to the ecliptic. According to Kepler, the ascendent node of Saturn is now at $22^\circ\ 49'$ of *Cancer*, & the inclination of its orbit with the ecliptic of $2^\circ\ 32'$; the b of *Jupiter* at $5^\circ\ 31'$ of Cancer, & the inclination of $1^\circ\ 20'$; the b of Mars at $17^\circ\ 50'$ of Taurus, & the inclination of $3^\circ\ 22'$; the b of Mercury at $14^\circ\ 47'$ of Taurus, & the inclination of $6^\circ\ 54'$.[2] In all these determinations, the Astronomers of our times agree very nearly: but they are quite different on the position of the solar equator: the more so as the observations of which serve us for this result, are not of a nature to provide correct determination of it. In the *Histoire de l'Académie Royale des Sciences de Paris pour l'année 1701*[3], the solar equator is deduced to make an angle with the ecliptic of $7^\circ\ 30'$, & in the Memoirs of the same year, it is said, that the Pole which Ursa Minor watches corresponds to the eighth degree of Pisces. By following these hypotheses, the solar equator is cut by the orbit

of Saturn, at an angle of $5^\circ\ 58'$
Jupiter $6^\circ\ 21'$
The Earth $7^\circ\ 30'$
Mars $5^\circ\ 49'$
Venus $4^\circ\ 10'$
Mercury $2^\circ\ 56'$

It is here the orbit of the Earth, which makes the greatest angle with the solar equator; namely of $7^\circ\ 30'$.

[It is easy to see what is the method to find the inclinations of the orbits with the solar equator; it does not differ from that *to find the inclinations that the Orbits make among themselves*, exposed above in the remark of § IV. Because knowing the solar node of the ecliptic, & the nodes of the planetary orbits with the ecliptic, the distance of the solar node to the other nodes, give a side in the spherical triangle to resolve: the angles that the solar equator, & the planetary orbits make with the ecliptic, are the two known angles in the same triangle; whence we find the third angle, which is the angle of inclination of the orbits with the solar equator.]

§ XXII. But as the position of the solar equator is quite uncertain; of such manner that, according to some, its inclination with the ecliptic does not surpass two degrees, we could perhaps without absurdity, pretend one such position, that its mean inclination with all the planetary orbits, is the least, in which condition we can satisfy by trying a great number of positions; thus, for example, in the preceding hypothesis the mean inclination of the orbits with the solar equator, is $5^\circ\ 11'$; but if we supposed that this equator makes with the ecliptic an angle of $3^\circ\ 22'$, & if its North Pole corresponds to 20° of Pisces, then the solar equator would be cut by the orbit

of	Saturn, at an angle of	$1^\circ\ 51'$
	Jupiter	$2^\circ\ 7'$
	The Earth	$2^\circ\ 4'$
	Mars	$3^\circ\ 22'$
	Venus	$0^\circ\ 20'$
	Mercury	$4^\circ\ 34'$

& the mean inclination of the orbits (which had been as much as $5^\circ\ 11'$) would be no more than $2^\circ\ 23'$. I do not know if we can not prefer this position of the solar equator, although supported on a pure conjecture, & founded *à posteriori*, to the other positions, based on

[2] Kepler, *Tabulae Rudolphinae*, Ulm 1627, pars secunda Tab. fol. 48, 54, 60, 66, 72.

[3] *Sur les taches du soleil*: Mém. Paris 1701, p. 104.

the spots of the Sun, by waiting that the Astronomers give us a more exact Astronomical method.

§ XXIII. By explaining above mechanically the action of the solar atmosphere on the Earth, & on the Planets, I have considered the material of the atmosphere as sloughed with more velocity than the bodies which it surrounds; it is not that our system demands it thus, but because this seems to me probable besides.

Now let, if we wish it, that the material does not slough itself more rapidly, & even that it sloughs itself more slowly, it will not be permitted to make the same effect on the orbits, by approaching them to the solar equator. In order to be convinced of this, we have only to resolve the movement of the material into two; the one parallel, & the other perpendicular to the direction of the Planet; & we see enough that this last acts always towards the Equator, not knowing how to fail to push the Planet towards this side.

§ XXIV. From the Planets we come to the Comets: I say that the planes of the Orbits of these will never shift sensibly their inclination with the solar Equator, as great as it be, either because they are almost always positioned entirely outside of the solar atmosphere (as truly the Moon is outside that of the Earth, & the fifth Satellite of Saturn outside of that of Saturn) or because they are not permitted to divert from their route because of the too great subtlety of the material of the atmosphere, which surrounds them during their almost entire revolution. It is true, that the Comets being near to their perihelion, there must approach a little to the solar Equator; but this time is hardly comparable with the rest of the time of the revolution, & it seems by the examples alleged above §§ XI. & XII. on the densities of the solar atmosphere that the density beginning one time to decrease, it decreases so rapidly that it loses at first sight nearly all entirely; all that indicates why the Comets, of which the distance to the Sun is during nearly all the time of the revolution as infinite, do not tend sensibly towards the Equator of the Sun. I would believe nonetheless easily, that the Orbits of the Comets from all the time of their existence are approaching a little the said Equator; which makes me lean further to this opinion, and that in the great number of Comets marked in the Ephemerides, it has seemed to me that the mean inclination of their Orbits with respect to the solar Equator, would not miss to be quite nearly 45°, if they were actually a little approximate; I have therefore collected the observations of several Comets which have appeared from some centuries; & in order to spare the pain of calculation, I have supposed that the said mean inclination with respect to the solar Equator is the same as that with respect to the ecliptic, their planes differ not much, & the differences of these two kinds of inclination being unable to fail of destroying themselves very nearly both; which makes thus that we have no need to be very scrupulous on the correctness of the observations, because their errors will destroy themselves of like strength probably. There is therefore the catalogue of the Comets.

Of the Comet of the year	1337,	the inclination to the ecliptic	32°	$11'$	$0''$
	1472		5°	$20'$	$0''$
	1531		17°	$56'$	$0''$
	1532		32°	$36'$	$0''$
	1556		32°	$6'$	$0''$
	1577		74°	$32'$	$45''$
	1580		64°	$40'$	$0''$
	1585		6°	$4'$	$0''$
	1590		29°	$40'$	$40''$
	1596		55°	$12'$	$0''$
	1607		17°	$2'$	$0''$
	1618		37°	$34'$	$0''$
	1652		79°	$28'$	$0''$
	1661		32°	$35'$	$50''$
	1664		21°	$18'$	$30''$
	1665		76°	$5'$	$0''$
	1672		83°	$22'$	$10''$
	1677		79°	$3'$	$15''$
	1680		60°	$56'$	$0''$
	1682		17°	$56'$	$0''$
	1683		83°	$11'$	$0''$
	1684		65°	$43'$	$40''$
	1686		31°	$21'$	$40''$
	1694		11°	$46'$	$0''$

The mean inclination is $43^\circ\ 39'$. It is therefore clear that the Comets have nearly no point of liaison at all with the solar Equator, & that they approach it only insensibly, & with an extreme slowness.

[§ XXV.– §XXVIII. continue discussion of the theory of the atmosphere.]