Mensura sortis ad fortuitam successionem rerum naturaliter contingentium applicata*

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§ 1. Whoever will have examined carefully the various anthropological tables assembled with immense labor, readily, from the great number of observations, he will have observed many laws of nature neither foreseen nor expected and I may have said the same concerning innumerable other things, if only the explorations had been arranged with equal diligence and likewise according to a different aspect of the surrounding things. There are two which come to be observed in regard to inquiries of this kind: namely accidental events, which we ascribe to chance, and the laws prescribed to chance itself decided out of the great number of events; while there may be countless proofs, which pertain here I shall submit a single example and I will choose the very one because it gives an opportunity from these little observations. When I examined again the various anthropological tables I have pursued the argument which treats of the proportion by which births in each sex are divided, but all admit a male offspring to prevail now unanimously; of the phenomenon of proclivity either it has happened by pure chance although by its nature equally to each sex or some measure itself will be by chance, by which more proclivity is returned to the male sex than to the other, clearly as the lot in the case of a proposed cast of two dice is said to have more tendency to the number seven than to the six; indeed it will explain that hesitation by far better, which will have determined the degree of probability for any prior event; in order that I may undertake this labor for myself, I have not hesitated; and I will investigate both hypotheses if God will have granted life and strength and I shall connect the emerging numbers to the tables which they offer for sale. According to the present circumstances of things, I shall run through the first hypothesis, by which the nature to the formation of each sex is assumed equally easy and favorable.

§ 2. Let the number of annual births have been $= 2N$, which thus I make even in order that the number of each sex is able to be the same; it is sought how much be the probability, that the number of boys may obtain a given or prescribed value; let that number $= m$; moreover the probability of anything is indicated by a fraction of which the numerator must have a ratio to the denominator as the number of favorable cases to the number of all cases, if with equal facility single cases occur; the maximum

*The measurement of risk applied to the chance succession of things occuring naturally.

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probability in this sense, which is able to be had, is expressed by unity and it indicates determined things, which are not able to not happen. But the sought probability is obtained easily from the theory of combinations, to be

$$= \frac{2N \cdot (2N-1) \cdot (2N-2) \ldots (2N-m+3) \cdot (2N-m+2) \cdot (2N-m+1)}{1 \cdot 2 \cdot 3 \ldots (m-2) \cdot (m-1) \cdot m} \times \frac{1}{2^{2N}}$$

where so in the numerator as in the denominator of the indefinite fraction there are as many factors to be taken as there are units in m. If however the case $m = 0$ is proposed, which namely the individual births are imagined to have given a girl, it is proven by itself the probability to be then $= \frac{1}{2^{2N}}$ and precisely the least by far even if the number will have been average for a year of births. With the number m increasing the probability increases and moreover the maximum is in the middle, where it is supposed $m = N$ and there is the same number of infants of each sex exactly; beyond the middle the probability decreases so that for equal distances from the middle the probability is equal, which has been noted from the nature of the coefficients[1] in the binomial raised to the $2N$ power, which our formula itself of the second paragraph establishes successively if the number m proceeds in the form of the natural numbers.

§ 3. Of the cases, of which we have made mention just now, more than enough observations of the equality of both sexes occur; therefore let us make for that case $m = N$ and thus our general formula exposed in the preceding paragraph will be of such kind

$$= \frac{2N \cdot (2N-1) \cdot (2N-2) \ldots (N+3) \cdot (N+2) \cdot (N+1)}{1 \cdot 2 \cdot 3 \ldots (N-2) \cdot (N-1) \cdot N} \times \frac{1}{2^{2N}}.$$

Thus therefore for just a few births that maximum probability is determined easily; where however the greater number of them is so much more difficult and it will happen more rarely that they should divide into two precisely equal classes by one's sex; nevertheless the probability near to the number of births increases with increasing N, which we shall demonstrate next: hence it will happen that an inequality between each sex, divided by the number of births, probably decreases, because it must be asserted equally concerning all observations, and they are of any birth whatsoever, by proceeding with a variable step, all Authors agree in regard to this; a smaller aberration to be presumed from many established observations than from fewer; but no one thus far, who I know, has shown the pattern, according to which those aberrations, with all the rest being equal, are diminished by repeated observations; this I shall deliver further down.

§ 4. The indefinite formula, which I have set forth in the preceding paragraph, produces this with the inevitable inconvenience that it requires insurmountable labor, if a large number of births will have been (also it ascends at Paris and London nearly to twenty thousand) although tables of logarithms may be summoned for help; but with regard to examples of this kind generally the word is of them most painstaking and likewise, discouraged by the enormity of the calculation, I have sought a quick path,

[1] *Translator's note*: The word is "unciarum." In mathematics at that time, the word "uncia" referred to the number prefixed to the powers of the variable(s) in the expansion of a binomial or multinomial.

which I shall describe somewhat carefully, seeing that in most other arguments it is advantageous.

In the first place, the analytic and indefinite expression of the preceding paragraph I have translated into another form more ordered and likewise more accommodated to our intent, certainly this

$$\frac{1}{2} \times \frac{3}{4} \times \frac{5}{6} \times \frac{7}{8} \cdots \frac{2N-5}{2N-4} \times \frac{2N-3}{2N-2} \times \frac{2N-1}{2N},$$

where there are as many factors to be received by its series as there are units in N, that is, in number of births halved. I have attacked the thing hereafter thus with this aforementioned transformation.

§ 5. Let be put, for the number of factors N, the product of all the factors $= q$ and afterwards let us put a new factor to arrive, namely $N+1$ to be put in the place of the number N; thus a new product for $(N+1)$ factors will happen

$$= \frac{2N+1}{2N+2} q = q - \frac{q}{2N+2};$$

Therefore as often as the number N is increased by unity, so much is the product diminished by the quantity

$$\frac{q}{2N+2};$$

that decrement is indeed very small, when the number N is assumed very great; hence it will be approximately

$$-dq : dN = \frac{q}{2N+2} : 1;$$

whence there appears

$$-\frac{dq}{q} = \frac{dN}{2N+2};$$

And by this differential equation, after it will have been integrated, in order that it will be permitted for determining the value q for a given number N, if only some initial factors in the formula of the preceding paragraph will have been actually multiplied among themselves; nevertheless, it pleases the other with the cause of greater accuracy, as if of the next rank, to add a little correction; therefore it should be observed the value $\frac{dq}{q}$ to have been deduced out of the change which rises if in place of N there is put $N+1$; but it may have been able to be brought forth by equal right out of the derived change which happens when in place of N there is put $N-1$ and then a slightly different differential equation is obtained, certainly

$$-\frac{dq}{q} = \frac{dN}{2N-1};$$

hence it is concluded rightly, it will be a more accurate differential equation if the mean denominator is taken between $2N+2$ and $2N-1$ or the equal of the half sum of them, that is, $2N+\frac{1}{2}$; therefore we shall enjoy a more accurate differential equation and satisfying beyond expectation

$$-\frac{dq}{q} = \frac{dN}{2N+\frac{1}{2}}.$$

The integration of that equation thus should be established so that, with some halved constant quantity to be added, it should become to some case; by which however it will have held many initial factors, therefore the formula will be more accurate for as many subsequent factors as many as it pleases; therefore let us put, because with the posited number of initial factors $= f$, the emerging product is $= A$; it will be had by the integration of the aforementioned differential formula

$$\log \frac{A}{q} = \frac{1}{2} \log \frac{2N + \frac{1}{2}}{2f + \frac{1}{2}},$$

or

$$q = A\sqrt{\frac{4f+1}{4N+1}}.$$

§ 6. The excellence of the aforesaid method will shine forth to a greater extent if an example is assumed of which the value of which is well known through itself. The indefinite formula is displayed, most similar with the preceding, certainly

$$\frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} \times \frac{4}{5} \cdots \frac{N-2}{N-1} \times \frac{N-1}{N} \times \frac{N}{N+1},$$

of which the value is evidently

$$= \frac{1}{N+1};$$

but let us see what the new method will indicate; let again the product of all factors be $= q$; if indeed a new factor arrives, the product will be

$$\frac{N+1}{N+2} q = q - \frac{q}{N+2};$$

whence now there arises

$$-dq : dN = \frac{q}{N+2} : 1$$

or

$$-\frac{dq}{q} = \frac{dN}{N+2};$$

Because if from the opposite the ultimate factor may be trimmed, there happens

$$-\frac{dq}{q} = \frac{dN}{N};$$

therefore with the mean denominator assumed again, we will make

$$-\frac{dq}{q} = \frac{dN}{N+1},$$

of which the integral, with the significance retained of the letters f and A used before, there will be

$$\log \frac{A}{q} = \log \frac{N+1}{f+1}$$

or

$$q = \frac{f+1}{N+1}A;$$

but certainly in that example there is

$$A = \frac{1}{f+1};$$

therefore it is simply and generally

$$q = \frac{1}{N+1},$$

thus in order that the method, in this example at least, indicates exactly what the thing is. Many others may be able to be added here concerning the thing, if the reasoning of our principle permits it.

§ 7. Whoever wishes to apply the simple equation exposed at the end of the fifth paragraph to any examples whatsoever, will discern the sensible aberration certainly with difficulty. But where the greater number of initial factors, indicated by f, actually will have multiplied and will have determined the number A, therefore will discover the sought product q more accurately for the entire number of factors N. I shall allege the example by the method itself as much as harmful; I shall put simply $f = 2$; thus there will be

$$A = \frac{1}{2} \times \frac{3}{4} = \frac{3}{8}$$

and there becomes

$$q = \frac{9}{8\sqrt{4N+1}};$$

let be put further simply $N = 6$ and thus there will be $q = \frac{9}{40}$. But the true product is

$$\frac{1}{2} \times \frac{3}{4} \times \frac{5}{6} \times \frac{7}{8} \times \frac{9}{10} \times \frac{11}{12} = \frac{231}{1024},$$

which differs from the other scarcely; but our formula will respond much more accurately, if the computation is prepared over a greater number f; I will put therefore $f = 12$ and exactly

$$\sqrt{4f+1} = 7;$$

thus the product becomes out of the initial twelve factors or

$$A = \frac{676039}{4194304};$$

whence

$$A\sqrt{4f+1} = \frac{4732273}{4194304} = 1.12826;$$

and exactly for the assumed number $f = 12$, it produces

$$q = \frac{1.12826}{\sqrt{4N+1}};$$

and with that value, whatever will have been the number N, and clearly without any sensible error when it will be permitted.

§ 8. It is understood from the general formula § 2. if the number of annual births will have been somewhat greater, the probability is almost null around the beginning and the end and to become a maximum around the middle although even then it may be exceedingly small for any specific case. But now it is further understood this same probability to decrease with increasing number of births and thus certainly that it may follow as nearly as possible a subduplicated inverse ratio of the begotten offspring; that theorem supports manifestly the calculations, which first seemed insurmountable from the front; in fact one computation of an example indicates immediately all the rest. I should wish it be observed in addition to be investigated as a rule especially, anything by lot is be able to be divided about the middle, where there is a small difference between one or other sex, while hardly it is able to happen that an inequality may cross certain limits, because experience itself testifies, if we consider variations, not inequalities. This is the reason, because the argument is more convenient to pursue from the middle toward the extremities than in turn.

§ 9. I will descend now to an example: I shall suppose a very great number of births, to be of such kind it is accustomed yearly in Paris and in London; I shall make $2N = 20000$ or $N = 10000$; for this example it is sought, in the nature of the hypothesis the tendency to be equally in the formation of each sex, how much be the probability that the number of boys is precisely 10000. No one surely will attempt the computation according to the lead of the formula exposed in the fourth paragraph, and much less will he recur to the formula in the third paragraph; indeed if we make use of the formula delivered at the end in the seventh paragraph, immediately we discover $q = 0.0056413$ or $q = \frac{1}{177\frac{1}{4}}$.
it rejoices with such probability, to which the thing will have been equal by lot with 176 gamblers; the expectation is very small, yet I reckoned smaller before the calculation established. If indeed in an average city the fecundity is put one hundred times less, there will be by the preceding paragraph, for a similar event an expectation ten times greater and exactly $=\frac{10}{177\frac{1}{4}}$.
This from the case of an equal division of births brought into the world with regard to each sex.

§ 10. Now let the value of the letter N be supposed to continue and to abate each variation in the value of the letter m or in the number of boys comprehended in the annual generation of men; namely the mind is to investigate the variation of probability, while the number m slowly and slowly, either in one or in another part, recedes from the number N. Therefore I will make successively

$$m = N \pm 1; \quad m = N \pm 2; \quad m = N \pm 3; \ldots \quad m = N \pm \mu,$$

where μ indicates the number by which boys are distant from the number N either in excess or in defect, because in both parts the probability decreases equally. But it is clear from the formulas employed §§ 2 and 3, the probability successively to be

$$\frac{N}{N+1}q; \quad \frac{N \times (N-1)}{(N+1) \times (N+2)}q; \quad \frac{N \times (N-1) \times (N-2)}{(N+1) \times (N+2) \times (N+3)}q \quad \text{etc.}$$

whence the general formula which expresses the probability, by which the number of boys becomes $= N \pm \mu$, now will be of such kind

$$\frac{N}{N+1} \times \frac{N-1}{N+2} \times \frac{N-2}{N+3} \times \cdots \times \frac{N-\mu+1}{N+\mu} \times q.$$

But the value of the letter q, by which this formula has been multiplied, is to this point (see § 7)

$$= \frac{1.12826}{\sqrt{4N+1}}.$$

§ 11. Behold now the table constructed according to the precept previous paragraph where again $N = 10000$ is assumed, in which the first column denotes the value of the letter μ, the other however the probability corresponding to it.

I.	II.	I.	II.
1.	$0.9999q$	26.	$0.9341q$
2.	$0.9996q$	27.	$0.9291q$
3.	$0.9991q$	28.	$0.9239q$
4.	$0.9984q$	29.	$0.9185q$
5.	$0.9975q$	30.	$0.9131q$
6.	$0.9964q$	31.	$0.9076q$
7.	$0.9951q$	32.	$0.9019q$
8.	$0.9936q$	33.	$0.8961q$
9.	$0.9919q$	34.	$0.8902q$
10.	$0.9900q$	35.	$0.8842q$
11.	$0.9879q$	36.	$0.8780q$
12.	$0.9856q$	37.	$0.8716q$
13.	$0.9831q$	38.	$0.8651q$
14.	$0.9804q$	39.	$0.8585q$
15.	$0.9775q$	40.	$0.8517q$
16.	$0.9744q$	41.	$0.8449q$
17.	$0.9711q$	42.	$0.8379q$
18.	$0.9677q$	43.	$0.8308q$
19.	$0.9641q$	44.	$0.8236q$
20.	$0.9604q$	45.	$0.8163q$
21.	$0.9565q$	46.	$0.8089q$
22.	$0.9524q$	47.	$0.8014q$
23.	$0.9481q$	48.	$0.7938q$
24.	$0.9136q$	49.	$0.7861q$
25.	$0.9389q$	50.	$0.7783q$

§ 12. I have placed this first small portion of the table to that end so that thence I might be able to deduce the solution of the not at all useless question, which I had conceived by reason; namely the two limits equidistant from the middle are sought from this law, in order that the probability is the same, the numbers of boys transcending or not transcending these limits.

The solution of this question requires, that there must be investigated how many terms of the table from the beginning toward the end must be added, that the double of the sum increased by the quantity q becomes $= \frac{1}{2}$; for the sum of the terms, beyond the middle position, gives the sum of the probabilities, that the number of boys not exceed the limit, and the same sum on the near side of the middle of the positions expresses the sum of the probabilities that the number of boys not descend below the opposite limit, it is doubled on account of the sums; finally to this double sum must be added the probability q itself for the case $\mu = 0$ or for the equality between each sex. According to the pattern of this precept the distance of each limit from the middle is discovered or $\mu = 47$ as near as possible. Since the sum of the forty-seven terms is $= 43.4606q$, the double of it $= 87.2812q$ to which if q is added besides there arises $88.2812q$. But we have discovered in paragraph nine $q = 0.0056413$, whence at last the final quantity produces 0.4980, which is smaller than $\frac{1}{2}$ by only a small amount. Because if certainly $\mu = 48$ is assumed, then the same final quantity is 0.5070 and moreover greater than $\frac{1}{2}$ interpolation gives approximately $\mu = 47\frac{1}{4}$; but the mutual distance of the limits will be $2\mu = 94\frac{1}{2}$. Therefore finally among 20000 annual births it will be equally probable, that the number of males may not stray outside the limits 9953 and 10047 than that it transgress those limits, if only the lot favor each sex equally. Yet the latter is very small amount more probable on account of a small neglected fraction.

§ 13. In like manner our problem will be solved for any other number N; certainly what brings new labor for any example whatever? A shortcut is given of the work; for I say the sought numbers μ follow as near as possible the subduplicated ratio of given number N, which certainly out of the aforementioned, if they had been examined more carefully, it is gathered easily, and the shortcut will be so much more accurate as the number of births will be assumed greater. Let the annual number of births in all France be assumed or $2N = 600000$, and exactly $N = 300000$, which number is thirty times greater than the preceding; now I say $\mu = 47\frac{1}{4} \times \sqrt{30} = 258.8$ and $2\mu = 517.6$. Thus therefore according to the hypothesis, which we are discussing, of an equivalent lot for each sex, the contest will be equal, either you must contend a greater difference of sex to be in France, or less.

Because if indeed for a moderately fruitful city the number of annual births is assumed $= 200$, that is, one hundred times fewer than what has been assumed in the preceding paragraph, the number μ will become ten times less or $= 47\frac{1}{4}/10 = 4.725$, and the two limits will not stand apart more than 9.45 from one another. Hence we understand, to be by a small amount the probability that the difference between each sex not exceed ten than that it exceed.

§ 14. Now it is pleasing to compute directly that latter example, that the truth of the shortcut used in the preceding paragraph shines forth; I shall set to the goal this other portion of the table for $N = 100$ to lead to the indefinite formula established in the tenth paragraph, in which it will suffice to have considered five terms for the natural numbers 1, 2, 3, 4, and 5, with this other goal I shall add the probabilities for the numbers 10, 15, 20, 25, 30, 35, and 40: the first column again by its order indicates these individual numbers, while the second column shows the corresponding probabilities; indeed the maximum probability, for the case of perfect equality between one or the other sex, that is, for $\mu = 0$, now I will indicate by q'.

I.	II.	I.	II.
1.	$0.9901q'$	10.	$0.3679q'$
2.	$0.9608q'$	15.	$0.1054q'$
3.	$0.9141q'$	20.	$0.01832q'$
4.	$0.8522q'$	25.	$0.001931q'$
5.	$0.7789q'$	30.	$0.0001235q'$

But there is nearly

$$q' = \frac{10}{177} \qquad (\S\ 9)$$

or more accurately

$$q' = \frac{1.12826}{\sqrt{401}} = 0.05634$$

by strength of the seventh paragraph and this value will be substituted with regard to each number.

§ 15. Now let the sum of the first five terms be obtained and it will be held $4.4979q'$; the double of this $= 8.9958q'$, to which q' must be added and thus $9.9958q'$ or 0.5631 will be obtained and indeed this quantity is greater than $\frac{1}{2}$; but if the first four terms are obtained the sum of them will be $= 3.7182q'$; the double of which $= 7.4362q'$; if q' is added now $8.4364q'$ or 0.4753 will be had but this quantity now is less than $\frac{1}{2}$; therefore by our short rule it matches well with the true value: thus therefore for the two hundred children if the contest happens, the sex is going to be more different than ten or not, I say the latter to be more probable and eventually to be an equal contest, if they are being offered 5631 against 4369: more correctly he will be able to offer boldly two hundred times a thousand thousands against one, if either sex will descend below 60.

§ 16. What has been said concerning the question in the twelfth paragraph, for which the limits for equiprobability, or the values μ according to the given number N, must be determined, the innate characters of it are that they are able to be converted hence new questions are formed, in which analysis may dominate. Let there be, for the sake of an example, two Gamblers contending by a game of dice between themselves continuously, each with anyone casting, exactly as the game will have brought forth either an even number or an odd, he will gain a single point, nor first is the thing limited, than when the one will have excelled the other by 94 points, the number of casts is sought after which it is equally probable, that the contest must be ended or not ended. The twelfth paragraph indicates the solution; certainly the number of casts will be $=$ 20000 or lesser by a small quantity on account of the neglected fraction to be added to the number 94. If for a gain of 94 points there is substituted fewer, just as of nine or ten points, 200 casts will be required by the power of the thirteenth paragraph. Nor do I see a more direct method, by which questions of this kind are able to admit a solution.

§ 17. The very small table in the fourteenth paragraph indicates not obscurely to us, by what law the probabilities decrease from the middle to the extremes; particularly furthermore it is apparent, all probabilities at an average distance from the middle so far to not vanish all at once since, if $\mu = 40$, the relative probability $= 0.0000007774q'$ and the absolute probability $= 0.000000004478$, which even if they are added with all probabilities for the succeeding terms but for with each small measure is able to be disregarded.

I must wish further the harmony be noted, which exists between the tables §§ 11 and 14, seeing that the probabilities in the first table for the numbers 10, 20, 30, 40 and 50 are expressed nearly with the same numeric coefficients, which in the other table with the ten times smaller numbers that is, for the numbers 1, 2, 3, 4 and 5, which indeed by no means with difficulty was able to be provided out of our theory. Certainly because the numbers of the second column in the table of the eleventh paragraph are not changed excessively, it follows thence the sum to be nearly ten times greater, seeing that ten times as many terms are to be taken and since on the contrary q' is just about ten times greater than q, it follows because the sums of 10, 20, 30, 40 or 50 terms is approximately the equal in table § 11. of the sum 1, 2, 3, 4 or 5 of the terms in table § 14 after evidently the values for q and q' had been substituted.

§ 18. Observations of this kind are able to come to use admirably well. Thus we understand the same one to be nearly an absolute probability, that among two hundred births the number of boys not ascend except to 70, what is among 20000 births, that it not reach except to 9700; nonetheless each probability is small that there is a surplus to extend the calculations beyond these limits, in the first case it is $\mu = 30$ in the other $\mu = 300$.

One thing is what I will add; certainly I shall not give the indefinite but the definite formula, which does not express very badly the numbers of each table, if only we remain within certain limits; I have discovered this formula by nearly the same method, where I have used in the fifth paragraph for discovering as nearly as possible the value q, which gives the probability for perfect equality between either sex. Let generally the half of all births be the number $= N$; and let again the number of boys be $= N \pm \mu$. I say the probability of this case to be nearly

$$= \frac{Q}{c^{\frac{\mu\mu}{N}}}$$

if only the number μ is not much greater than $\sqrt{N}$. Moreover Q is the probability for the case of equality between each sex and c is the number of which the hyperbolic logarithm is unity, or $c = 2.718$.

§ 19. In order that it is permitted to distinguish the strength of this simplest formula, expressed by one term, I will return to the very small table § 14 in which $N = 100$ and $q' = Q$, and I will compute the numbers of the second column to the regarded of that formula; thus aberrations will become known immediately.

I.	II.	I.	II.
1.	$0.9901q'$	15.	$0.1057q'$
2.	$0.9610q'$	20.	$0.01819q'$
3.	$0.9143q'$	25.	$0.001864q'$
4.	$0.8528q'$	30.	$0.0001124q'$
5.	$0.7797q'$	35.	$0.000003924q'$
10.	$0.3691q'$	40.	$0.00000007774q'$

Out of the comparison it appears the aberrations hardly to be sensible and where by a little they start to become relatively more sensible, there the total absolute probabilities nearly to vanish. It will be permitted certainly by the short rule that as long as the number μ does not exceed the number $3\sqrt{N}$.

Likewise we will balance with a scale the formula for another and certainly with a greater value of the letter N by far. Let there be again $N = 10000$ that we are able to collect diverse cases with the table exhibited in the eleventh paragraph and directly with computation; thus the value Q will be the same in so far it is indicated in the table by the letter q; but I will put for μ successively the numbers 10, 20, 30, 40, and 50: with these set the corresponding probabilities are discovered $0.9901q$; $0.9608q$; $0.9141q$; $0.8522q$; and $0.7889q$; certainly the table itself of the eleventh paragraph gives $0.9900q$; $0.9604q$; $0.9131q$; $0.8517q$; and $0.7783q$; which numbers surely do not agree so much with precision between them.

§ 20. I shall add one example of the rule by a well-known excellence, by which it is clear how everywhere the calculus must be established for any numbers whatsoever.

In the excellent work concerning these things of the most deserving man by far, *Johann Peter Süssmilch*, from the second edition third part[2] several tables have been attached, in which page 17 we see, that in the year 1758 within Zeeland Province[3] 3533 little boys and 3805 little girls had been born; whence the number of births $= 2N = 7338$ and $N = 3669$; certainly because the number of boys was 3805, there will be had $\mu = 136$; hence

$$\frac{\mu\mu}{N} = 5\frac{151}{3669} = 5.04;$$

therefore the probability, by which the number of boys is precisely $= 3805$, in this example is indicated by the formula

$$\frac{Q}{c^{5.04}} = 0.0006628Q,$$

and that value is so much more accurate, because the number μ has ascended very little beyond the number $2\sqrt{N}$ therefore the probability for the case of equality between each sex is to the probability by which the number of boys exceeds that equality precisely by the number 136 as 10000000 to 0.0006628; certainly the absolute probability is had for the last case if for Q or q the value 0.009313 defined in the seventh paragraph is substituted of course; in this manner that absolute probability must be $= 0.000006173$; but I say, because there is able to be uncertainty in this very small value, it alone to consider carefully the last two numerical figures. Thus therefore we see the absolute probability to be able to be determined for every case without that through the intermediate cases, let us advance and this is finally because I maintained principally, when I undertook these inquiries.

§ 21. What things have been said thus far are purely analytic, seeing that for the creation of little boys or little girls the extraction of tickets either black or white from an urn is able to be substituted if only the ticket is restored into the urn before a new

[2]J. P. Süssmilch "Die Göttliche Ordnung in den Veränderungen des menschlichen Geschlechts, aus der Geburt, Tod und Fortpflanzung desselben erwiesen." Berlin 1741 (1st edition)

[3]Denmark

extraction happens; then, if they have been deposited into the urn with an equal number of tickets of each color, we will have, what has been the thing to us. Because if certainly we suppose the black tickets to prevail in number, we have cut into the other hypothesis, by which it is assumed nature to incline more to forming the masculine sex than the feminine. That other argument, which is prior as it contains the simple case in itself, is not able to not present new labors not yet explored to myself; whatsoever it be of trouble I shall undertake with nearest leisure.