SUR DEUX MÉMOIRES DE D'ALEMBERT L'UN CONCERNANT LE CALCUL DES PROBABILITÉS L'AUTRE L'INOCULATION

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ON THE PROBABILITIES

Mr. d'Alembert just published his *Opuscules mathématiques*. There are in this compilation two memoirs which it is not impossible to reduce to the ordinary language of reason.

The one has for object *the calculus of the probabilities*; the calculus of which the application has so much importance and extent. This is properly *the physico-mathematical science* of life. The other treats of the advantages or disadvantages of *inoculation*.

The examination of some particular cases has made suspect to Mr. d'Alembert a hidden vice, in the general rule of the analysis of chances.

Here is this rule: *Multiply the gain or the loss which each event must produce, by the probability that there is that this event must arrive. Add together all these products, by regarding the losses as negative gains; and you will have the expectation of the player; or, that which amounts to the same, the sum that this player must give before the game, in order to begin to play start to finish.*

This rule appears simple and totally conformed to good sense. However if we suppose that *Pierre* and *Jacques* play at *heads or tails*, on the condition that if Pierre brings forth *heads* at the first toss Jacques will give to him an écu; that if Pierre brings forth heads only on the second toss, Jacques will give to him two écus; if on the third, four écus; if on fourth, eight écus; if on the fifth, sixteen écus and thus in sequence according to the same progression, and if we seek by the present rule the expectation of Pierre, or that which he must give to Jacques in order to begin to play with him start to finish, we find an infinite sum.

Now, besides that an infinite sum is a chimera, who is it who would wish to give, says Mr. d'Alembert, not this sum, but a sum moderate enough, in order to play this game.

We respond to Mr. d'Alembert, that if the stake of Pierre is found infinite, it is that we have made the tacit and false assumption that the game must endure always and that all the tosses can take place.

Mr. d'Alembert replies that in the number of cases, those where *heads* never arrives and *tails* always arrives is found as another and that it has its value;

That if we claim that *heads* arrives necessarily after a finite number of throws, at least this number is indeterminate;

That any sum which we assign for the stake of Pierre, it will be contestable;

That we can support only that it is indeterminate, because in the end a man can propose this game to another, and that one accept it;

That if Pierre would give fifty écus to Jacques and if we fix at one hundred the number of tosses to play, it would be necessary in order that Pierre recovered this sum by playing,

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that heads came only at the seventh toss, a risk that assuredly a person would not wish to incur.

An able geometer (this is, I believe, Mr. Fontaine[1]) has remarked that the stake of Pierre was neither infinite nor indeterminate; that whatever richness which we supposed to the two players, they would not have the wherewithal to play one hundred tosses and that thus the stake of Pierre cannot exceed fifty écus.

Mr. d'Alembert says next to this that in order to recover this loss of fifty écus, it would be necessary that heads arrive only at the seventh toss; that there are odds of 127 to 1 against that it will arrive earlier and that Pierre will lose his stake in whole or in part;

That there is no wise man who gave 78125 livres for a ticket of a lottery composed of one hundred twenty-seven bad tickets against one good, of ten millions;

And if we have regard, he adds, to the wrong that a loss of 78125 livres would make to the fortune of a player; then the loss will no longer be purely and simply proportional to the expected sum.[2]

Whence Mr. d'Alembert concludes that, when the probability of an event is quite small, it is necessary to treat it as null, and not multiply it at all by the expected gain, however considerable that it be, in order to find the expectation or the stake, that is to say that then there has no sum in the world which can compensate the risk.

He adds that in playing at *heads or tails*, the combinations where the *heads* and the *tails* will be the most mixed will be also the most frequent. He intends by being mixed, not happening a great number of times in sequence, and he regards these cases as more probables and more possibles than the others.

He distinguishes a *metaphysical possible* and a *physical possible*; he understands under the first all that which implies no contradiction, however rare or extraordinary that it be. Under the second, all that which is common, frequent and according to the daily course of events. Thus, according to this idea, it is a *metaphysical possibility* to bring forth a rafle of six with two dice one hundred times in sequence; but this is a *physical impossibility*.

But if in the ordinary usage of life, it is necessary to regard as null a very small probability, we demand to Mr. d'Alembert where is the term where it will cease to be null and where it will be able to be treated as something. Besides if the probability which is of a thousandth is not ignored, how to estimate that which is a little greater? If the value of the probabilities vary, what is the law of this variability: And if the geometer has no response at all to these questions, what becomes the analysis of the probabilities?

Mr. d'Alembert sends back the solution of these difficulties to the knowledge of the rare and frequent cases, that is to say to experience.

There will be therefore some exactitude in the analysis of chances only after some centuries of observation? It is true, responds Mr. d'Alembert.

Here is another of his ideas. It is that in *even or not*, in *heads or tails*, the past trials are something to the following, and that, consequently, the more *heads* will have arrived in consecutive times, the more there will be appearance that tails will arrive the toss to follow. — *And what is the law of this increasing in appearance?* — I know nothing of it. — *And the law of combinations that does it become?* — That which it might.

An assumption of the analysis of probabilities that Mr. d'Alembert attacks next, is that *in the number of possible combinations those which bring forth many times in sequence the same event are as possible as each of the others, taken in particular.*

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[1]See *Recueil des Mémoires* de M. Fontaine.

[2]All this is not extracted from the text of d'Alembert. Diderot follows the reasoning, but does not take the same numbers.

If we represent *heads* by a and *tails* by b, he denies that the case $aaaaaa$, etc. is as possible as the case $aababa$, etc.

But if the possibility varies among the cases, what rule is made on that? — I know nothing of it. — *Will we count for something the possibility of the cases where the same event takes place three, four, five times in sequence?* — It will be necessary to see. — *Where to begin? . . . where to end? . . . When one will have begun, what law will the probabilities follow? If the law varies, what will be its variability? Without these preliminaries known, no analysis.* — This can be.

Mr. d'Alembert himself had demanded in the word *heads or tails* in the *Encyclopedia*, how much one must wager to bring forth *heads* in two trials.

The ordinary response, is that the stake is 3 against 1.

That of Mr. d'Alembert, is that it is 2 against 1.

In order to prove that it is 3 against 1, we say: there are four different combinations, *heads-heads; heads-tails; tails-heads; tails-tails*. The first three make wins; the last alone makes a loss; therefore the stake must be 3 against 1.

Mr. d'Alembert responds: If *heads* arrives on the first toss, the game is ended, we do not play a second. The combinations *heads-heads* and *heads-tails* are reduced therefore to one. There are only three possible combinations, two which make wins and one which makes a loss: therefore the stake must be of 2 against 1.

He believes that the manner in which we reason in order to prove that the stake is 3 against 1 is paralogistic, and that its paralogism is increased still if the wager is to bring forth *heads*, not in two tosses, but in one hundred tosses played in sequence; because, says he, then we treat the combination which would bring forth *heads* one hundred consecutive times as possible as another also; that which it is not.

We say to Mr. d'Alembert: But the probability to bring forth *heads* at the first toss is one half, and this case is favorable.

The probability to bring forth *tails* at the first toss is also one half, and this case is null.

And in the case where we bring forth *tails* at the first toss, the probability to bring forth *heads* at the second toss is one half multiplied by one half, or one fourth, and this case is favorable.

The probability to bring forth *tails* at the second toss is also one half multiplied by one half, and this case is unfavorable.

The sum of the favorable probabilities is therefore to those of the unfavorable probabilities as $\frac{1}{2} + \frac{1}{4}$ is to $\frac{1}{4}$, or as 3 to 1.

In this reasoning, says Mr. d'Alembert, we treat the first toss as the second. Now this must not be done, because the first toss is *certain*, and the second is only *probable*. He adds that, besides, this manner to estimate the probabilities is subject to all the difficulties which are born of the assumption of an equal probability for all the possible combinations, a supposition contrary to the ordinary course of things.

We insist and we say to him: The combinations *heads; tails-heads; tails-tails;* are the only possibles. — *Agreed.* — but the probability of bringing forth *heads* at the first toss is equal to that of bringing forth *tails* at the first toss. — *I swear it.* — Now, the probability to bring forth *tails* at the first toss is double of that to bring forth *tails* at the first toss and *heads* at the second, or *tails* on the first toss and *tails* on the second. — *I swear it.* — Therefore . . . — *I deny the consequence*.

The argument is not in form. The mean term, the term of comparison is not the same in the *major* and in the *minor*. This mean term in the *major*, it is the *probability* to bring forth *tails* at the first toss, before having played this first toss; in the *minor*, it is the *probability* to bring forth *tails* at the first toss, compared to the probability of bringing forth *heads* or *tails* at the second toss. Now *probability* supposes here the first toss played and *tails* brought forth, therefore to bring forth *tails* at the first toss is no longer *probability*, but *certitude*. In a word, there is this difference between *heads* and *tails*, at the first toss, than *heads* brought forth, no more second toss; *tails* brought forth, second toss necessary. And then, why would not the toss *tails-heads* not be a little more probable than the toss *tails-tails*? *Tails-tails* is twice in sequence the same event. If the probabilities of *tails-heads* and of *tails-tails* are unequals, then I swear, says Mr. d'Alembert, that the ratio of the stakes will be neither of 3 to 1, as we wish it, nor of 2 to 1, as I have believed it. — *What is it therefore?* — Perhaps incommensurable, inappreciable. — *And this supposed, what becomes the analysis of the probabilities?* — This is not my affair. That which I perceive, is that the general rule by which we determine the ratio of the probabilities, is not correct; that a theory satisfying the probabilities supposes the solution of many perhaps insoluble questions, as to assign the ratio of the probabilities in the cases which are not, or that it is necessary to regard as being not equally possibles; to fix when the probability is small enough in order to be treated as null; finally, to estimate the stake according as the probability is more or less great.

Mr. d'Alembert claims that the combination $aaaaaa$ is less possible than the combination $ababab$. I swear that setting aside all physical cause, which favors one or the other, this proposition appears to me still void of sense.

I carry the same judgment of the solution which he gives of the problem of the stake of the one who proposes to bring forth *heads* in two tosses and of the one who accepts this game. Nothing is more false than these stakes being as 2 to 1 or in some other ratio than the one of 3 to 1.

As he has made it an affair of dialectic, it is necessary to argue against him, and to show him the little foundation of the distinction of the *possible* case and of the *certain* case, by separating these ideas from the solution.

If a player has equal expectation, in playing a single trial, to obtain either nothing or a cup of gold, it is clear that the value of his trial is half of the cup of gold.

If a player has equal expectation, by playing a single trial, to obtain either a helmet or a cup of gold, or whatever sort of advantage it be, it is clear that the value of his trial is the half of these advantages; thus, in the example proposed of the case of the helmet and of the cup) it is the half of the helmet, plus the half of the cup.

This put, if Pierre and Jacques play at *heads and tails*, and if Jacques accords two tosses to Pierre in order to bring forth *heads*, we see what must be the stake of Pierre, and what is the stake of Jacques.

Let any unknown quantity be the sum of the stake of Pierre and of the stake of Jacques.

When Pierre is at the point of playing his first toss, his expectation is the same to the entire sum of the stakes, and to a second toss.

Therefore the value of his expectation is the half of the entire sum of the stakes, plus the half of a second toss.

But what is the value of this second toss for Pierre?

Since this second toss gives to him an equal expectation to the entire sum of the stakes and to nothing, his value is half the entire sum of the stakes, and the half of its value of the fourth of the entire sum of the stakes.

Therefore, when Pierre is at the point of playing his first toss, the value of his expectation is the half of the entire sum of the stakes, plus of the fourth of the entire sum of the stakes, or else of three-fourths of the entire sum of the stakes.

Therefore, the value of the expectation of Jacques is of a fourth of the entire sum of the stakes.

Therefore, the value of the expectation of Pierre is the value of the expectation of Jacques as three-fourths to one fourth, or as 3 to 1.

Therefore the stake of Pierre must be to that of Jacques as 3 to 1.

The same reasoning is applied to the case where player A proposes to player B an écu, if he brings forth *heads* on the first toss; two écus, if he brings forth *heads* only at the second toss; four écus, if he brings forth *heads* only at the third toss; eight écus, if he brings forth *heads* only at the fourth toss; sixteen écus, if he brings forth *heads* only at the fifth toss; and thus in sequence according to the same progression.

I say: when B is at the point of playing his first toss, his expectation is the same to an écu and to a second toss.

Therefore, the value of his expectation is the half of an écu, plus the half of a second toss.

But what is the value of this second toss?

Since this second toss gives to him equal expectation to two écus and to a third toss, therefore the value of this second toss is an écu, plus the half of a third toss; and the value of the half of this second toss, a half-écu, plus the fourth of a third toss.

Therefore, when Pierre is at the point of playing his first toss, his expectation is the half of an écu, plus the half of an écu, plus the fourth of a third toss.

But what is the value of this third toss?

Since this third toss gives to him equal expectation to four écus, plus to a fourth toss, therefore the value of this third toss is two écus, plus the half of a fourth toss; and the value of the quarter of this third toss, of the half of one écu, plus the eighth of a fourth toss.

Therefore, when Pierre is at the point of playing his first toss, his expectation is the half of an écu, plus the half of an écu, plus the half of an écu, plus the eighth of a fourth toss.

But what is the value of this fourth toss?

Since this fourth toss gives to him equal expectation to eight écus and to a fifth toss, therefore the value of this fourth toss is four écus, plus the half of a fifth toss; and the value of the eighth of this fourth toss of a half-écu, plus the sixteenth of a fifth toss.

Therefore, when Pierre is at the point of playing his first toss, his expectation is the half of an écu, plus the half of an écu, plus the half of an écu, plus the half of an écu, plus the sixteenth of a fifth toss.

And thus in sequence.

Whence we see that the expression of the expectation of Pierre will contain always a half-écu, plus a portion of the second toss; or a half-écu, plus a half-écu, plus a portion of the third toss; or a half-écu, plus a half-écu, plus a half-écu, plus a portion of the fourth toss; and thus to the toss at infinity.

Therefore, we suppose that A and B play during all eternity.

And under this assumption, the toss at infinity never being able to take place, we see that the expectation of the players or their reciprocal advantage tends without ceasing to equality, but never arrives. Whence we see that this solution returns to the idea that I have given of an equal game, when I have said that an equal game was the one where it had odds of one to one at each trial, and where, the more one would play trials, the more the ratio of the winning trials to the losing trials would approach to the ratio of equality, sometimes giving this ratio, ordinarily deviating from it, either to lower, or to higher.

When Mr. d'Alembert has distinguished the first toss, which he calls certain, from the second toss, which he calls probable, he has not seen that there is neither question of probability to play nor certitude to play, but of the claims or reciprocal expectations of the players before playing; of that which would return to each of them, if they would not wish to play, but to share the stake; and that these claims, anterior to the first toss by their nature, would admit no distinction of probability or of certitude.

It is not of it of two trials as to an infinite number of trials, thus:

If a player has equal expectation, by playing a single trial, to obtain either 0 or P, it is certain that the value of his trial $= \frac{P}{2}$. This is evident.

If a player has equal expectation, by playing a single trial, to obtain either P or ϕ, in a word some sort of advantages as it be, it is certain that the value of his trial $= \frac{P}{2} + \frac{\phi}{2}$.

This put, if Jacques accords at *heads or tails* two tosses to Pierre in order to bring forth *heads*, see what must be the stake of Pierre and what the stake of Jacques.

Let P be the sum of the stake of Pierre and of the stake of Jacques. I say that the claim of Pierre, when he is at the point of playing his first toss, $= \frac{3P}{4}$; consequently, that of Jacques $= \frac{P}{4}$, and the stake of Pierre is to that of Jacques as 3 to 1. Because, when Pierre is at the point of playing his first toss, his claim is the same to P and to a toss which assures to him equally either 0 or P.

Now, a toss to which we have the same claim as to P and which assures equally either 0 or $P = \frac{P}{2}$.

Therefore, when Pierre is at the point of playing his first toss, his claim is the same to P and to $\frac{P}{2}$.

Now, a claim which is the same to P and to $\frac{P}{2} = \frac{P}{2} + \frac{P}{4} = \frac{3P}{4}$.

Therefore, when Pierre is at the point of playing his first toss, his claim $= \frac{3P}{4}$; therefore that of Jacques $= \frac{P}{4}$; therefore the stake of Pierre to that of Jacques is as 3 to 1.

The same demonstration is applied to the case where player A proposes to player B an écu if he brings forth *heads* at the first toss, 2 écus if he brings forth *heads* only at the second toss, 4 écus if he brings forth *heads* only at the third toss, 8 écus if he brings forth *heads* only at the fourth toss, 16 écus if he brings forth *heads* only at the fifth toss, and thus in sequence following the same progression of them.

I say: the claim of B, when he is at the point of playing his first toss, is the same to 1 écu and to a second toss.

Therefore, whatever be the value of this second toss, the claim of B when he is at the point of playing his first toss $= \frac{1}{2} + \frac{\text{a 2nd toss}}{2}$.

But this second toss assures to him equally either 2 écus or a third toss; therefore the value of this second toss $= 1 + \frac{\text{a 3rd toss}}{2}$.

Therefore the claim of B, when he is at the point of playing his first toss $= \frac{1}{2} + \frac{1}{2} + \frac{\text{a 3rd toss}}{4}$.

But this third toss assures to him equally 4 écus or a fourth toss; therefore the value of this 3rd toss $= 2 + \frac{\text{a 4th toss}}{2}$.

Therefore the claim of B when he is at the point of playing his first toss $= \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{\text{a 4th toss}}{8}$.

But this fourth toss assures to him equally 8 écus or a fifth toss; therefore the value of this 4th toss $= 4 + \frac{\text{a 5th toss}}{2}$.

Therefore, the claim of B when he is at the point of playing his first toss $= \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{\text{a 5th toss}}{16}$.

The paradox of Mr. d'Alembert consists, when he has distinguished the first toss, which he calls *certain*, from the second toss which he calls *probable*, in not having seen that there is neither question of *probability* nor of *certitude*; but of the claim of the player before playing; of that which would return to him if he would not wish to play, and that this claim to P and to any other quantity of which the chance gives to him an equal alternative admits no other distinction.

SOME OBSERVATIONS ON THIS MEMOIR

The analysis of probabilities can be considered as an *abstract science* or as a *physico-mathematical science*.

Under the first aspect the problems resolve themselves in the head of the geometer, as they would be resolved in the divine understanding. A period which has no endpoint tends at each instant to give an infinite value to the smallest finite quantities. The results must never astonish. As the combination is executed without cessation, there is nothing which it cannot bring forth. Time is tantamount to all. Suppose to an atom of matter an absolute hardness; place this atom on a block of marble large as is the universe; animate it by the least degree of weight; with this feeble effort and time, it will arrive to the center of the globe. With time, all that which is possible in nature is. If eternity multiplied the least degree of possibility, the product will be equal to the most enormous possibility multiplied by the instant which follows.

Under the second aspect, this is a science restrained in small ways, at an experience of a moment, a being which passes as lightning and which yields all to its duration.

All mathematical science is full of those falsities which Mr. d'Alembert reproaches in the analysis of probabilities. Whence are born the *incommensurables*? the impossibility of the *rectifications* and of the *quadratures*? This is the fable of Daedalus. Man has made the labyrinth and he is lost.

In the problem of the two players at *heads or tails* of which the solution revolts the mind at first sight, all absurdity is in the names of the players. Instead of *Pierre* and of *Jacques*, say: *Oromaze* and *Arimane* play without ceasing, and the infinite stake will be just and the game equal. For what is an equal game? One where there are odds of one against one at each trial, and where consequently one uninterrupted sequence of trials tends without ceasing to render the number of losing trials equal to the number of winning trials.

When you say: A and B play, you institute A and B playing during all eternity: this is a permanent state. Your solution is eternal, and when you say: *Pierre* and *Jacques* play, you restrict it to an instant. The expression *play* is indefinite in the first case; in the second, to the contrary, it is determined.

The question was *physico-mathematical*, and your solution is abstract; the question supposed some infinite beings, and your solution is applied to some finite beings, whence it follows from it that we have made to enter by calculation a multitude of casts which never could be, an imaginary advantage, a chimerical duration, of the *sums* and a game without interruption and a life without end.

In order to remain in the *physico-mathematical* and to accord the demand with the response, here is how it was necessary to propose the problem.

Pierre and Jacques (two men) *engage themselves in a game all their life, in such game and under such conditions; what must be their stakes?*

Then it is necessary to find the mean expression of the duration of a trial. Young, we play more lively than old, the morning more lively than at the end of the day. This is a labor: we can scarcely play than the times that we would work. Each deduction, time given to repose and to needs and taken by the distractions and maladies, the remainder of the day which we employ either to a labor or to a continued game will be a little thing.

It is necessary to have the probable duration of life of the elder, because it is necessary that they both live, it is necessary that they have each the greatest sum which is possible to lose in each game.

But if the condition is of *to play all the life*, I know not if the expression of the times will not be a variable quantity, because at each trial lost or won it will be necessary to recommence, and then other values of the duration of the trial, of the game, of life, of the stakes, and then who knows if this expression *two players play* will not remain unlimited; will it not suppose a permanent and eternal state, and if the question will not return again, in this way, into the class of *abstractions*? I suspect this expression *play* of which we make perhaps a permanent state in the solution, and which is a momentary state in the application, to be in part the cause of all these differences which Mr. d'Alembert establishes between the successive tosses and the mixed tosses, because we have no sooner extended the duration to infinity than this difference disappears,, and it diminishes in measure as the number of the trials or as the notion of the duration of the game is increased; this is a consideration which is worth the pain that we pause on it. When we say in the announcement of a problem A and B *play*, perhaps we suppose either that they *play* always or that they play only a single trial.

Mr. d'Alembert says that in the number of cases, the one where *tails* arrives always, and *heads* never, is found as another. . . *Yes, as another that one specifies likewise. Now, in order to bring forth a specified trial among an infinity of different other trials, an infinity of tosses is necessary; an infinite duration, and the players A and B can no longer be men.*

Mr. d'Alembert says that if we claim that *heads* arrives after a certain number of tosses, at least this number is undetermined and that whatever sum which we assign to the stake of Pierre it will be contestable. . . *This is true;* and the reason that M. d'Alembert has not seen, is that there is not and that he can have no game where some physical causes introduce a secret inappreciable inequality. We believe by playing with a six-faced die, to play a game to six equal chances, this which is false; it would be necessary that the center of gravity be rigorously at the center of mass, that which is impossible in an instant; that which will be possible in an instant and would cease to take place in the following instant.

A single die gives at least six unequal chances. Thence this distinction that experience indicates between one case and another.

We have much shaken the dice-box, the dice neither are stirred not at all nor on the table of trictrac as if they were perfect. The physical cause has this effect; thence the cards *are bent*, the trials *are ben*t and so much of fine observations of the players of profession.

Now, the effect of physical causes changes perpetually. Soon they tend to bring forth a like event many times in sequence, soon another event, but also many times in sequence.

Mr. d'Alembert responds to the ingenious solution of Mr. Fontaine, that it would be necessary, in order that Pierre recover his stake, that *heads* arrived only at the seventh toss and that there are odds 127 against 1 that it will arrive earlier; but what does it signify, if a single toss can be worth to Pierre 127 times his stake and more?

If a man does not set 78125 livres on a lottery ticket which can be worth 10 millions, but on which there are odds 127 against 1 that it will be worth nothing, it is that there are some games which are not at all made for men and some men who are not at all made for the game.

The games in which men risk the least part of their happiness are not made for them.

Kings and men of exorbitant fortune are not made for the game.

Kings risk nothing, and those who play against them risk all.

Men who play with a great fortune can lose it against an unfortunate who has only an écu in his pocket.

Mr. d'Alembert says that *when the probability of an event is quite small it is necessary to treat it as null.* This proposition, advanced generally, as it is there, is false and contrary to the practice of players and merchants.

Those who make a fortune at the game and in affairs, have no other superiority over the others but to discern a small probability and to remove it from their rivals. At length, those who neglect the small advantages ruin themselves.

It is not at all that there is a small advantage when it is repeated; it is that there is no probability so small that has not its effect with time; it is that, in every game, perhaps it would be necessary to be subject to a certain number of trials and to increase the stakes according to a certain law; it is that it is necessary that this observation not be without foundation, seeing truly some players play not at all against a man who has only one trial to play and that others increase their stakes in measure as they lose.

Does Mr. d'Alembert wish to say that it is prudent to not risk a great sum in a game where the probability is very small, whatever be the gain proposed? I am of his opinion; but what does this do to the analysis of games of chance?

I add that in *heads or tails*, that in *even or not*, as in *dice*, the trials which have preceded do something to the trial which comes to follow. If I judge this proposition without any regard to some secret physical cause which determines an event to take place rather than another, I do not find it with sense.

Is it not of these two trials, such as of two men who have to pass a forest where they must endure a certain number of trials by gun, but on the condition that if the first who will pass is killed, the second either will pass not at all or will pass without peril, and that if the first is not killed, the second will pass and will incur the same peril as his predecessor; it is certain that one of these men will be to the other useful in order to pass first.

We finish these observations with some examples of men who are not too rare. These are some wise folk who always fail, and some madmen who constantly succeed. It is necessary to wish that the last die promptly and that the first live long, so that the chance of this evil game which we call life, and which one has made us play in spite of us, changes for the first and has not the time to change for the second. If a drunk man walks himself a long time on the edge of a precipice, it is necessary that he fall.