Absent

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ABSENT. (*Calculation of the probabilities.*) When Mr. Nicolas Bernoulli, nephew of the celebrated Jacques & Jean Bernoulli, sustained at Basel, in 1709, his thesis of doctor in law; as he was a great geometer, as well as jurisconsultant, he could not be prevented from choosing a matter which admits of Geometry. He took therefore for the subject of his thesis, *de usus artis conjectandi in Jure*; that is to say, *on the application of the calculus of probabilities too the matters of Jurisprudence*, & the third chapter of this thesis treats of the time where *an absentee must be reputed for dead.* According to him, he must be counted such, when he has twice more odds that he is dead than living. Suppose therefore a man departs from his country at the age of twenty years, & we see, according to the theory of Mr. Bernoulli, in what time he can be counted dead.

According to the Tables given by Mr. Deparcieux, of the Académie Royale des Sciences, of 814 persons living at the age of 20 years, there remain of them, at the age of 72 years, only 271, which is very nearly the third of 814; therefore there are dead the two-thirds from 20 to 72, that is to say, in 52 years; therefore, at the end of 52 years, there are twice more odds for dead than for life of a man who is absent & who disappeared at age 20 years. I have chosen here the Table of Mr. Deparcieux, & I have preferred it to the one which Mr. Bernoulli seems to have served himself, satisfying myself to apply here his reasoning: but I believe our calculation too strong on this occasion in a certain regard, & too feeble to another; because 1°. on the one hand, the Table of Mr. Deparcieux had been made on some annuitants of tontines who, as he himself notices it, live customarily more than the others, because one ordinarily applies to the tontine only when one is well enough constituted to be flattered with a long life. On the contrary, the odds are that a man who is absent, & who for a long time has not given news of himself to his family, is at least in misfortune or in destitution, which, combined with the tiredness of travels can hardly fail to shorten the days. 2°. On the other hand, I don't see that it is sufficient that a man is reputed dead, when the odds are only two against one that he is dead, especially in the case in question. For when there is a question to dispose of the wealth of a man, & to divest him of it without other motive than his long absence, the law must always suppose his certain death. This principle appears me so evident & so just, that if the Table of Mr. Deparcieux was not made out of some people who live customarily a longer time than the others, I would believe that the absentee must be reputed dead only in the time where there remain no more of the 814 persons aged 20 years, that is to say, to 93 years. But, since the Table of Mr. Deparcieux would be, in this case, too favorable to the *absentees*, one will be able, it seems there to me, to make a compensation, by taking the year where there remains only the quarter of the 814 persons, that is, about 75 years. This question would be easier to decide, if one had some Tables of mortality of travelers: but these Tables fail us again, because they are very difficult, & perhaps impossible in implementation.

Mr. Buffon has given at the end of the third volume of his Natural History, some Tables of the length of life, more exact & handier than those of Mr. Deparcieux, in order to resolve the problem in question, because they had been made for all the men without distinction, & not for the annuitants only. However these Tables could be perhaps again a little too favorable to the travelers, who must generally live less than the other men. But, since they are less than the others, instead of taking three-fourths, as we have done in the Tables of Mr. Deparcieux, it would be good to take only 5/6 of them, or perhaps 7/8 of them. The calculation of it is easy to make; it was sufficient for us to have indicated the method. (*M. d'Alembert*)

Besides the solution of this problem supposes another theory out of the moral problem of the events which one has followed to the present. This new theory is from Mr. Buffon, & we are going put the reader in a state of satisfying himself on the present question of *the absentee reputed for dead*, by indicating to him the principles that he could follow. It is constant that, when the question is to decide by an assumption of the well-being of a man, who has against him only his absence, it is necessary to have the greatest possible certitude that the assumption is true. But how to have this greater possible moral certitude? Where to take this *maximum*? How to determine it? Here is how Mr. Buffon wishes that one take it, & one cannot doubt that his idea is very ingenious, & gives the solution of a great number of embarassing questions, such as those of the problem respecting the sum which a player A must wager at heads or tails against a player B which would give to him an écu, if to him B would bring forth tails on the first toss; two écus, if to him B would bring forth tails at the second toss; four écus, if to him B would bring forth still tails at the third, & thus in sequence; because it is evident that the stake of A must be determined out of the greatest possible moral certitude that one can have, that B will not pass a certain number of tosses; this which makes the question return in the end, & give to him some limits. But one will have, in the case of the absentee, the greatest possible moral certitude of his death, or of an event in general, by the one where a number of mens would be great enough in order that none feared the greatest misfortune, which must arrive however infallibly to one among them. Example: take ten thousand men of the same age, of the same health, &c. among whom there must certainly die one of them today: if this number is not yet great enough to deliver entirely from the fear of the death of each of them, take twenty of them. Under this last assumption, the case where one would have the greatest possible moral certitude that a man would be dead, this would be the one where of these twenty thousand living men, when he is absented, there would remain of them no more than one.

Here is the route that one must follow here & in all other similar conjectures, where humanity seems to require the most favorable assumption. *This Addition is from Mr. DIDEROT.*

We have believed to must add to this article, drawn from the first edition of the Encyclopedia, the following reflections:

I. The new Theory which Mr. Buffon has proposed, does not at all appear to us to be admitted.

1.° It is inexact in itself, since it tends to confound two things of different nature, probability & certitude.

2.° It can serve only to simplify the calculation, in the case where the result, to which this hypothesis would lead, would differ only in an insensible manner from rigorous results: thus, it must not be employed to resolve any of the difficulties which can be raised out of the same principles of the calculus.

3.° One is permitted, in the calculus, to employ as absolute an approximate determination, only for a quantity of which one knows a value very little different from the real, & of which the real value is uncertain; now here the value of certitude is determined & equal to unity. Thus, there is no reason to suppose in the calculus, neither any probability equal to unity, nor certitude equal to a fraction little different from unity.

4.° When one is permitted to neglect a quantity, it is always by the reason that one regards it as null, with respect to that which one wants to know, & this is that which can not take place here. All the time that the probability of an event is very little different from unity, the probability of the contrary event is a very small quantity. Thus, I can well regard as being equally probable the events of which the probabilities $\frac{10^{10000}-1}{10^{10000}}$ & $\frac{10^{10001}-1}{10^{10001}}$ are very little different.

But it is not the same of the contrary events, of which the probabilities $\frac{1}{10^{10000}}$, $\frac{1}{10^{10001}}$ are in the ratio of one to ten.

5.° A *maximum* of probability is an expression which can not be understood by mathematics; the *maximum* of the probability would be 1, & it can not be attained. But one can fix a *minimum* of probability, that is to say, a probability below of which, for example, it can be permitted neither to condemn a man, nor to strip him of his wealth, nor to repute him dead. This idea is absolutely opposed to that which Mr. Buffon proposes, & it is the sole which one must admit.

II. In the questions relative to the wealths of the absentees, it is necessary to examine separately the following three hypotheses:

That where the *absentee* would return at the end of a certain time, that where the *absentee* will never return, but where the period of his death is known, & that where one ignores this period in perpetuity.

This put, we will demand first according to what principle one must regulate the administration of the wealth of the *absentee*, in order that, in the case where he would return, he had experienced no injustice.

It is clear that on his return, he would take back his claims on all his wealth, & that the question is therefore only to determine at what period one cease to leave them in sequestered, & to permit to his heirs to divide them, & to dispose of them, in giving, for their surety, the convenient guarantees, & at the end of which time one can even excuse them formalities.

For the 1st case, one sees that, in the moment where the life of a man is uncertain, one does an injustice to him, in the case where he would live, in permitting his heirs to dispose of his wealth, & that, if he is dead, one does one to his heirs in depriving them of the succession. Let therefore a be the probability of risk to which one you will expose an *absentee*, in permitting to distribute his wealth b at the end of a number n of years of absence; the value of the loss to which he would be exposed will be expressed by ab, & if u is the probability that he is living, his risk will be expressed by $\overline{uab}$.

Now, what is the loss of the heir, if one refuses to put it in possession? The wealth is here always b: let $\overline{1-r}$ be, the probability that he will always enjoy it, without being obliged to render it, that is to say, the probability that the absentee will not return; $1-r.b$ is the wrong done to the heir.

But this wrong is not irreparable, since, if one renders b to him, increased by interest, the year after he will have nothing lost; let therefore u' be the probability that he will not die in the year; $\overline{1-u'}\cdot\overline{1-r}\cdot b$, will express the risk that one makes him incur, by not remitting to him the disposition at the beginning of this same year; thus, in order to deliver the wealth of the *absentee* to the same heirs, in requiring a guarantee, it is necessary that $\overline{1-r}\cdot\overline{1-u'} > ua$. The case where one can cease to require the formalities of guarantees, will be resolved likewise, with that exception that a must be then greater. One can, without much error, suppose $1-r = 1-u+\frac{1}{2}u$: in effect $1-r$ is the probability that the *absentee* will never return; but the probability that he is dead is $1-u$, & one can suppose that, if he is living, there is as much to wager that he will return, as to wager that he will not return.

This 1st condition is not sufficient; it is yet necessary that, in the case where one would know one day the fixed period of the death of the *absentee*, there resulted of the dispositions made of his wealth, no injustice in regard to his heirs.

It is clear that the wealth must be divided definitely, as if the succession were opened on the day of his death. Thus, suppose that b is, in a given period, the portion of the wealth which heir A demands, & that, by the death of A, this portion would pass to B, the wrong done to A is here as above $\overline{1-r}\cdot\overline{1-u'}\cdot b$. In order to evaluate the wrong done to B, one will find:

1.° That a can express the danger to which one exposes him;

2.° That this danger takes place only as far as A would die before the *absentee*; therefore, if u'' expresses the probability that A dies before the *absentee*, one will have $u''ab$ for the risk to which B is exposed, if one gives a part of the inheritance to A. Thus, it will be necessary, in order to leave to A the disposition of his portion, that $\overline{1-r}\cdot\overline{1-u'} > u''a$.

This condition must be fulfilled in all the combinations which can take place in the order of mortality of the different heirs.

Suppose finally that there is a question to regulate the division of the wealth, by having equally regard to the justice in the assumption that the *absentee* never returns, & that one must be ignorant of the period of his death. We know, 1.° the period where one can, without injustice, make the division of his wealth. Let b be the wealth; since there is $1-u$ to wager that he is dead, it will be necessary first to take a part $\overline{1-u}b$.

2.° If, from the uncertainty of his lot until this period, there is for one or two other periods $A \cdot B$ some variations in the order of the succession, it will be necessary, if $1-u \cdot x$, $1-u \cdot \overline{x+z}$ represent the probability of the death in these periods, to divide $\overline{1-u} \cdot xb$, as in the period A; $\overline{1-u}z \cdot b$, as in the period of the distribution. The rest can be distributed provisionally, but always on the condition that, if a change in the order of succession occurs, a part of this remainder proportional to the probability that dead has not arrived before this change, will be distributed as it would have been, if one had learned that his death is arrived after this change. Finally, when the probability of the death of the *absentee* will have surpassed this *minimum* of moral certitude of which we have spoken above, it will be necessary to distribute definitely all the wealth, following the same rule[1]: whence there results that it is the only period in which the formalities of guarantees, &c. must be eliminated.

These principles suffice to resolve all the questions which can present themselves on this object; they limit themselves to this very simple rule, to suppose proportional to the probability of life & of death of the *absentee*, the wrong which results for him or for his heir, on the assumption that he is dead or living in each period that one considers, & to await, for each irrevocable decision, that the probability of death be beyond the *minimum* of probability, for which it can be permitted to deprive a man of his claims. (*See the article* PROBABILITÉ.) The questions relative to the distribution of the successions which can fall due to an *absentee*, to the claims of his creditors on his wealth, &c. must be resolved by the same principles.

We ourselves are limited here to some general principles, because their application in practice, depends on the laws of each country respecting the guarantees, on the prescription, on the order of succession, on the more or less of freedom of testimentary dispositions. Thus, in each nation, in order to make, respecting this object, a law conform to reason & to justice, it would be necessary, after having resolved, in all cases, the questions which can present themselves, to seek to deduce from all the results which one could obtain by the calculus, some general & simple rules, which could exhibit to commit some injustices, which in some cases nearly impossible to suppose. (*M.D.C.*)

[1] *See*, respecting the general principle to distribute the sums proportionally to the probability of the claim, *the article* PROBABILITÉ.