Correspondence on the Genoise Lottery

Leonhard Euler and Frederick the Great, King of Prussia[*]

Second series

Frederick II to Euler
Potsdam, 17 August 1763

A man named Griethousen coming from Holland sent me a plan of a lottery, which he thinks to establish in the land of Cleves, in order to aid this province in relieving itself from debts that it is seen obliged to contract during the troubles of the last war. I have well wished to communicate to you this plan enclosed, in order that you would examine it in all its detail, and would show me your sentiment, if you find it solid and equitable, in order to be agreeable to the public, and in order that the aforesaid province find the relief that the author makes to hope of it. I await the report that you will make for me of it, in which you will well want for me to make the pleasure of joining the aforementioned plan with your remarks. I would want moreover that you would not make a scandal of it again, nor that there transpires something from it out of season with the public, so much on those who consider the plan, as on its author. On this I pray God, that he has you in his holy guard.

Frederick

Euler to Frederick II
[Berlin] 19 August 1763

Sire,

After having examined by order of Your Majesty the plan of the lottery of Mr. de Griethousen, I find that the advantages that the state could hope from it, could be again more considerable than the author does not ensure, and could even climb to 8 million florins, as I believe to have demonstrated in the adjoined reflections. All things come back to knowledge if one could well expect whether this lottery be full, the sum of 100 fl. for each ticket [per year], and of 1000 fl. that the continuation during 10 years requires, will be always a very considerable object for most of the world, so that the distribution of the 50000 tickets will seem almost impossible if one does not know by experience, that such lotteries succeed well enough in Holland where the credit consolidated the confidence of the public. No one will ever learn anything from me concerning this project.

I dare to profit on this occasion by presenting to Your Majesty my very humble and very respectful thanks for the gracious insurances that Your Majesty has pleased well to bestow on my son.[1] I am penetrated by it most sharply and I die with the most profound respect.

Sire

Plan of a lottery of 5 classes, by Euler

Plan of a lottery of 5 classes each of 50000 tickets, of which all must be drawn in one year and repeated in 10 consecutive years.

	wager for a chance	revenue	number of chances	sum of the prizes	remainder in deposit
I. Class	10 fl.	500000 fl.	8000	292880	207120 fl.
II. Class	15 fl.	750000	8000	508967	241033
III. Class	20 fl.	1000000	8000	771480	228520
IV. Class	25 fl.	1250000	8000	1047208	202792
V. Class	30 fl.	1500000	8000	1524465	-24465
		5000000	40000	4145000	855000

After the drawing of these 5 classes, the remainder of 855000 fl. is used for a 6th class to make the prize of $28\frac{1}{2}$ fl. each for 30000 tickets, if there are as many who have won nothing in the 5 classes. This is here an incertitude, on which I have the following thoughts:

1. If all prizes in the 5 classes fell on different tickets, so that no one wins two, there would be only 10000 who would enter into the 6th class, of which each would receive $28\frac{1}{2}$ fl. that which making 28500 would leave a profit of 570000 fl. for the entrepreneurs or for the cash-box.

2. If all prizes in each class fell onto the same 8000 tickets, there would be 42000 without gain, of which 30000 would profit from the benefice of the 6th class, and 12000 would have absolutely nothing also the profit of the entrepreneur would vanish.

3. Now neither one nor the other of these two cases will probably ever exist, and one is able to suppose nearly enough that there will be ordinarily the middle that is to say 26000 tickets without gains, so that the 6th class will contain only that many tickets instead of 30000 and consequently the profit of the entrepreneur can be reputed 114000 fl.

4. This profit will become more considerable by the § 19 N.B. where the tenth part of each great lot of 1000 fl. and above must be shared among the 9 companions of the same parcel and which, consequently, will be excluded from the 6th class. As there are in the 5 classes 328 such shares or premiums, there could be 9 times as many, that is to say, 2957 who would not compete in the 6th class. But as many of them will be excluded from the same, in reckoning the number of participants in the 6th class will be diminished by the half 1500, and therefore the profit of the treasury must be estimated at 156750 fl.

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[1] His eldest son, Johann Albrecht, who since 6–12–1756 was an ordinary member of the Academy of Berlin, received since 1763 an annual rent.

5. The following years this profit will become more considerable because those who have won 1000 fl. and beyond in the preceding years will be in the future always excluded from the resource of the 6th class, although their shares will win nothing more.

6. The author sets this profit to the treasury by year at 250000 fl. which sum could be much too great, but as it is the chance, on which this sum depends, which could equally become so much greater as smaller, one would not count for certain. However one must regard it always as a very considerable thing.

7. This money lying in the treasury, is able to be employed during the 10 years, and providing interest at 5 percent as the author supposes: but at the end of this time he does not say what must become of this same capital, which must increase however to 2500000 fl. This sum which seems must be a real profit for the treasury is not included in the 7 millions, which is supposed to remain to the profit of the state.

8. There is again another benefit resulting from the proposed association where those who wish to profit, paying for their shares 10 percent beyond the ordinary wager, which makes 10 fl. per year on each share, and this surplus must be shared beyond 10 years among those of the partners who will have lost pro rata on their loss. During the times the sum is able to be placed at interest, which will fall to the profit of the treasury.

9. Beyond this, one deducts from each gain 10 percent which makes 500000 fl. per year, which sum, with those of the preceding articles will be put at interest at 5 percent and the author supposes that all this could increase to 1000000 per year of which it collects the interest, which would accrue all the years. Now he does not say to what this interest is employed all the years, if this is first to the profit of the country in order to pay the debts, or if they must remain in the treasury, in which case they could well be employed to produce new interest.

Frederick II to Euler
Potsdam, 20 August 1763

I thank you for the communication of some remarks that you have duly wanted to make on the plan of a lottery to establish in Cleves, which I have asked you to address, and that you have sent back to me in the sequel to the letter of the 19 of this month. It will be with pleasure that I embrace the occasions, where I can give to you the proofs of my contentment by reason of the difficulties that you yourself have given to the subject. On this I pray to God, that he have you in his holy care.

Frederic