APPENDICES TO CORRESPONDENCE WITH HUYGENS

J. HUDDE

The following four appendices to No. 1446 are taken from the notes of Hudde. These illustrate the method of solution Hudde used in solving the problems discussed with Huygens.

No. 1447[1]
J. Hudde to Christiaan Huygens
[1665]
Appendix I to No. 1446.

Solution of a question relating to the Advantage and to the Disadvantage of two players.

A and B play when nothing had been set, and stipulate that the one who casts tails will set a ducat, and that the one who casts heads will draw a ducat, under the condition that A will cast first. One demands the advantage of B? Response $\frac{1}{6}a$.[2]

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Prepared by Richard Pulskamp, Department of Mathematics and Computer Science, Xavier University, Cincinnati, OH. .

[1] T. V. page 463-465. This is the first of four pieces in the hand of Hudde. Probably all had been communicated later by Hudde to Huygens.

[2] In the solution which follows, Hudde admits the interpretation of Huygens on the manner in which the game must end, that is to say he supposes that the game does not end before one stake had been made on one side or the other. It is therefore clear that this piece must be posterior to the Letters No. 1434 and 1445. Moreover Huygens himself is occupied on this question, because one finds in his Adversaria, on the date of 15 July 1665, a solution of the same problem, formulated as follows:

A and B cast in turn at heads or tails, under the condition that the one who casts tails will set each time a ducat, but that the one who casts heads will win each time a ducat, so long as there will be found something in the game. And A will cast first, when nothing has yet been set: and the game will not be ended before something had been set, and one will play until that which all is anew withdrawn. One demands how much A will lose in the manner. It makes $\frac{1}{6}$ of a ducat.

It is therefore very probable that the problem in question has been posed later by Huygens to Hudde, who will arrive to the same result in this piece.

$a =$ a ducat

Advantages and disadvantages of B.

x	When A must cast without that anything has been set. Therefore When B must cast, without that anything has been set, it is $-x$.	$x = \frac{z-x}{2}$. Therefore $3x = z$.
z	When a ducat has been set by A, and when B must cast.	$z = \frac{a-y}{2}$
$-y$	The disadvantage of the coup, when by A and by B there has been set *one* against *one*, and when A must cast.	$-y = \frac{z-a+q}{2}$. Therefore $-2y = z-a+q$.

Note.
If one would wish to deny this corollary[3] the question is untraceable, except by a progression.[4]

Therefore
When there has been set 2 against 2 and when A must play, the disadvantage is anew $-y$.

When now one sets instead of z and of q the values that one has found, one obtains

$$-2y = \frac{a-y}{2} - a + \frac{-2y+a}{2}$$

and it comes after reduction
$-4y = -3y + 2a - 2a$
or by adding $3y$ on the two sides
$-y = 2a - 2a$. Therefore $-y = 0$.
But z was $= \frac{a-y}{2}$. Therefore
$z = \frac{1}{2}a$. Therefore $3x = \frac{1}{2}a$ and
$x = \frac{1}{6}a$.
Quod erat demonstratum.

q	When 2 ducats have been set by A and one by B, and when B must cast.	$q = \frac{-2y+a}{2}$

No. 1448[5]
J. Hudde to Christiaan Huygens
[1665]
Appendix II to No. 1446.

Advantages and disadvantages of B

x	When A must cast, when nothing has been set	$x = \frac{z-x}{2}$, $3x = z$
z	When a ducat has been set by A and when B must cast	$z = \frac{a-y}{2}$
$-y$	When a ducat has been set by A and one by B, and when A must cast	$-y = \frac{z-a+q}{2}$
q	When two ducats have been set by A and one by B, and when B must cast.	$q = \frac{-y+a-r}{2}$
$-r$	When 2 against 2 have been set and when A must cast.	$-r = \frac{q-a+s}{2}$
s	When 3 against 2 have been set and when B must cast	$s = \frac{-r+a-t}{2}$
$-t$	When 3 against 3 have been set and when A must cast	$-t = \frac{s-a+v}{2}$
v	When 4 against 3 have been set and when B must cast	$v = \frac{-t+a-w}{2}$
$-w$	When 4 against 4 have been set and when A must cast	$-w = \frac{v-a+b}{2}$, &c.

No. 1449[6]
J. Hudde to Christiaan Huygens
[1665]
Appendix III to No. 1446.

Progression in the question, when the one who casts heads draws only one ducat.

[5] T. V. page 466-467. This is the second of four pieces in the hand of Hudde.
[6] T. V. page 468-469. This is the third of four pieces in the hand of Hudde.

- $-y$
  - $+\frac{1}{2}z$
    - $+\frac{1}{4}a$
    - $-\frac{1}{4}y$
      - $+\frac{1}{8}z$
        - $+\frac{1}{16}a$
        - $-\frac{1}{16}y$
          - $+\frac{1}{32}z$ {1
          - $-\frac{1}{32}a$
          - $+\frac{1}{32}q$ {2
      - $-\frac{1}{8}a$
      - $+\frac{1}{8}q$
        - $-\frac{1}{16}y$
          - $+\frac{1}{32}z$ {3
          - $-\frac{1}{32}a$
          - $+\frac{1}{32}q$ {4
        - $+\frac{1}{16}a$
        - $-\frac{1}{16}r$
          - $+\frac{1}{32}q$ {5
          - $-\frac{1}{32}a$
          - $+\frac{1}{32}s$ {6
  - $-\frac{1}{2}a$
  - $+\frac{1}{2}q$
    - $-\frac{1}{4}y$
      - $+\frac{1}{4}z$
        - $+\frac{1}{16}a$
        - $-\frac{1}{16}y$
          - $+\frac{1}{32}z$ {7
          - $-\frac{1}{32}a$
          - $+\frac{1}{32}q$ {8
      - $-\frac{1}{8}a$
      - $+\frac{1}{8}q$
        - $-\frac{1}{16}y$
          - $+\frac{1}{32}z$ {9
          - $-\frac{1}{32}a$
          - $+\frac{1}{32}q$ {10
        - $+\frac{1}{16}a$
        - $-\frac{1}{16}r$
          - $+\frac{1}{2}q$ [7] {11
          - $-\frac{1}{2}a$ [8]
    - $+\frac{1}{4}a$
    - $-\frac{1}{4}y$
      - $+\frac{1}{8}q$
        - $-\frac{1}{16}y$
          - $-\frac{1}{2}s$ [9] {12
          - z [10] {13
          - q {14
        - $+\frac{1}{16}a$
        - $-\frac{1}{16}r$
          - q {15
          - s {16
      - $-\frac{1}{8}a$
      - $+\frac{1}{8}q$
        - $-\frac{1}{16}r$
          - q {17
          - s {18
        - $+\frac{1}{16}a$
        - $-\frac{1}{16}t$
          - s {19
          - v {20

$-y = -\frac{1}{2}a + \frac{2}{4}a - \frac{3}{8}a + \frac{6}{16}a - \frac{10}{32}a + \frac{20}{64}a$ &c. $= 0$. QED

No. 1450[11]

J. Hudde to Christiaan Huygens

[1665]

Appendix IV to No. 1446.

A and B having taken each 3 tokens, playing etc. One demands the chance of B.[12] The chance of B is $x = \frac{bz+cy}{b+c}$. Therefore $bx + cx = bz + cy$.

In order to find bz

When B has yet 2 tokens

$$z = \frac{br + cx}{b + c} \text{ Therefore}$$

$$bz + cz = br + cx$$

When B has no more than one token

$$r = \frac{cz}{b + c}$$

whence in multiplying by b

$$\left.\begin{aligned} br &= \frac{bcz}{b+c} \\ cx &= \frac{bcx + ccx}{b+c} \end{aligned}\right\} \text{by addition}$$

Therefore

$$bz + cz = \frac{bcz + bcx + ccx}{b + c}$$

This equation, being reduced, gives

$$z = \frac{bcx + ccx}{bb + bc + cc} \text{ multiplied by } b$$

$$bz = \frac{bbcx + bccx}{bb + bc + cc}$$

In order to find cy

When B has four tokens

$$y = \frac{bx + cq}{b + c} \text{ Therefore}$$

$$by + cy = bx + cq$$

When B has 5 tokens

$$q = \frac{by + ca}{b + c}$$

and in multiplying by c

$$\left.\begin{aligned} cq &= \frac{bcy + cca}{b+c} \\ bx &= \frac{bbx + bcx}{b+c} \end{aligned}\right\} \text{by addition}$$

Therefore

$$by + cy = \frac{bbx + bcx + bcy + cca}{b + c}$$

This equation, being reduced, gives

$$y = \frac{bbx + bcx + cca}{bb + bc + cc} \text{ multiplied by } c$$

$$cy = \frac{bbcx + bccx + c^3a}{bb + bc + cc}$$

Therefore the preceding

$$bx + cx = \frac{2bbcx + 2bccx + c^3a}{bb + bc + cc}$$

which equation, being reduced, gives

$$x = \frac{c^3a}{b^3 + c^3} \text{ quod erat Demonstrandum.}$$

$1 \quad \frac{ca}{b + c}$

$2 \quad \frac{c^2a}{b^2 + c^2}$

$3 \quad \frac{c^3a}{b^3 + c^3}$

Hinc colligo progressionem[13]

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[11]T. V. page 470-471. This is the fourth of four pieces in the hand of Hudde.

[12]The concern here is for the following problem: A and B possess at the beginning of the game each three tokens. The chance of A, at each coup, to win a token from B is represented by $\frac{b}{b+c}$, that of B to win a token of A by $\frac{c}{b+c}$. The game does not finish before one of the players has monopolized all the tokens. One demands the mathematical expectation of B, by representing by a the value of each token.

One will note the narrow resemblance of this problem with the last of the problems posed by Huygens at the end of his Treatise.