VII. DE SECTIONE ANGULI

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With entering the Year 1707, I have fallen into a Method by which, with a given equation of this form.

$$ny + \frac{nn-1}{2 \times 3}Ay^3 + \frac{nn-9}{4 \times 5}By^5 + \frac{nn-25}{6 \times 7}Cy^7 \text{ \&c.} = a,$$

Or of that,

$$ny + \frac{1-nn}{2 \times 3}Ay^3 + \frac{9-nn}{4 \times 5}By^5 + \frac{25-nn}{6 \times 7}Cy^7 \text{ \&c.} = a;$$

where the quantities A, B, C, &c. represent Coefficients of the preceding Terms, I have determined the Roots according to this manner.

With posed $\quad a + \sqrt{aa+1} = v$ in the first case.

$\qquad\qquad a + \sqrt{aa-1} = v$ in the second.

There will be $y = \frac{1}{2}\sqrt[n]{v} - \frac{\frac{1}{2}}{\sqrt[n]{v}}$ in the first case.

$\qquad y = \frac{1}{2}\sqrt[n]{v} + \frac{\frac{1}{2}}{\sqrt[n]{v}}$ in the second.

Moreover those solutions have been inserted into Philosophical Transactions, Number 309, for the months *Jan. Feb. March* of the same year.[1]

Even now in which it will have been examined by what artifice those formulas may have been discovered, by this without doubt the approach to the demonstration of the following Theorem will be clear.

Let x be the Versine[2] of any Arc.

$\quad t$ the Versine of another Arc.

$\quad 1$ the Radius of the Circle.

And let the first Arc to the latter be as 1 to n, Then, with the two equations assumed which it is permitted to call cognate,

$$1 - 2z^n + z^{2n} = -2z^n t$$
$$1 - 2z + zz = -2zx.$$

And by expunging z, the Equation will arise by which the Relation between x & t is determined.[3]

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[1]"Aequationum Quarundam Potestatis Tertiae, Quintae, Septimae, Nonae, & Superiorum, ad Infinitum Usque Pergendo, in Terminis Finitis, ad Instar Regularum pro Cubicis Quae Vocantur Cardani, Resolutio Analytica," Philosophical Transactions, Vol. 25, No. 309, pp. 2368-2371.

[3]*Translator's note*: Here Moivre's famous result $(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$ is implicit. Hence put $x = 1 - \cos\theta$ and $t = 1 - \cos n\theta$. Solving the first equation for z^n and the second for z we have respectively

$$z^n = \cos n\theta \pm i\sin n\theta \quad \text{and} \quad z = \cos\theta \pm i\sin\theta$$

The result then follows immediately.

COROLLARY I.

If the latter Arc is a Semicircumference, the Equations will be

$$1 + z^n = 0$$
$$1 - 2z + zz = -2zx.$$

from which if z is expunged, the Equation will arise by which the Versines of the Arcs are determined which are to the Semicircumference, once, three times, five times, &c. assumed, as 1 to n.

COROLLARY II.

If the latter Arc is a Circumference, the Equations will be

$$1 - z^n = 0$$
$$1 - 2z + zz = -2zx.$$

from which if z is expunged, the Equation will arise by which the Versines of the Arcs are determined which are to the Circumference, once, twice, three times, four times, &c. assumed as 1 to n.

COROLLARY III.

If the latter Arc is 60 Degrees, the Equations will be

$$1 - z^n + z^{2n} = 0$$
$$1 - 2z + zz = -2zx.$$

from which if z is expunged, the Equation will arise by which the Versines of the Arcs are determined which are to the Arc of 60 degrees

$$\text{through a } \left\{ \begin{array}{ccccc} 1, & 7, & 13, & 19, & 25 \\ 5, & 11, & 17, & 23, & 29 \end{array} \text{ \&c.} \right\} \text{ multiplication}$$

as 1 to n.

If the latter Arc is 120 Degrees, the Equations will be

$$1 + z^n + z^{2n} = 0$$
$$1 - 2z + zz = -2zx.$$

from which if z is expunged, the Equation will arise by which the Versines of the Arcs are determined which are to the Arc 120 Degrees

$$\text{through a } \left\{ \begin{array}{ccccc} 1, & 4, & 7, & 10, & 13 \\ 2, & 5, & 8, & 11, & 14 \end{array} \text{ \&c.} \right\} \text{ multiplication}$$

as 1 to n.

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