SOLUTION TO THE FIRST PROBLEM OF HUYGENS

BENEDICT SPINOZA

The solution of this problem is found in the short work "Reeckening van kanssen" which was reprinted in the *Benedicti de Spinoza, Opera quotquot reperta sunt*, Volume III, pp. 248–252. The following is prepared from the French translation in the collected works of Christiaan Huygens, Vol. I, pp. 29–31.

First Problem

A and B play one against the other with 2 dice with the following condition: A will have won if he casts 6 points, B if he casts 7 points. A will make first one single coup; next B 2 coups the one after the other; next anew A 2 coups, and thus consecutively, until one or the other has won. We demand the ratio of the chance of A to that of B. Response: as 10355 is to 12276.

In order to respond to this question, I divide it, according to the second rule of the Art of thinking of Mr Descartes[1]

First Proposition

B and A play one against the other with 2 dice with this condition, that B will win if he casts 7 points and A if he casts 6 of them, provided that each makes two coups consecutively, and that B casts the first. Their chances are B $\frac{14256}{22631}$, A $\frac{8375}{22631}$.

Analysis and Demonstration

Let x be the value of the chance of A, and that which one must put, or the stake, be called a, then the chance of B is worth therefore $a-x$. It seems likewise that, under this supposition, each time that the turn of B returns the chance of A will be anew x, but every time that it is the turn of A to cast, his chance must be greater. We designate by y that which this chance is worth then. Since therefore B must cast the first and since 6 coups of 2 dice, among the 36 which there are in all, are able to give to him 7 points, we have found that out of the two times where he is permitted to cast them, he has (after reduction of the ratio) 11 chances to a, or to win, and 25 which are lacking to him, namely, which make the turn of A return.[2] Consequently A, when B begins to cast, has 11 chances to 0, or to lose, and 25 chances to have y, that is that this will be his turn to cast. This is worth to A $\frac{25y}{36}$, but since we have supposed that the chance of A is worth x at the beginning, we have therefore $\frac{25y}{36} = x$, and hence $y = \frac{36x}{25}$. In order to find the value of y again in another fashion, it is certain that when A must cast, he has 5 chances to a, or to win, because he has 5 chances of the 36 which are able to give him 6 points; all well counted we have established that, in two coups, A has 335 chances to a, and 961 which make the turn return to B, that is

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[1]See "Discourse on Method."

[2]We find indeed $11 : 25$ for the relation of the chances in remarking that B has 6×36 chances to win on the first coup and 30×6 to win on the second coup, the total number of chances being 36×36.

which give him x. This is worth $\frac{335a+961x}{1296}$. As that must be $= y$ and as we have found above $\frac{36x}{25} = y$, it is necessary therefore that $\frac{335a+961x}{1296}$ is equal to $\frac{36x}{25}$, whence we deduce $x = \frac{8375a}{22631}$, this which is the chance of A, and consequently the chance of B will be worth $\frac{14256}{22631}a$. The chance of A is therefore to that of B as 8375 to 14256 and reciprocally that of B to A as 14256 to 8375. That which it was necessary to demonstrate.

Second Proposition

A plays against B as it is described in the problem. Their chances are A $\frac{10355}{22631}$, *B* $\frac{12276}{22631}$.

Since A has 5 chances to a, or else to win, and 31 chances to be lacking, that is to be found in the case of the first proposition, that which is worth to him $\frac{8375}{22631}a$: he has therefore 5 chances to $\frac{22631}{22631}a$ (in order that I reduce all to the same denominator), and 31 chances to $\frac{8375}{22631}a$.

22631		8375		22631	
5	Multiply	31	Multiply	36	Multiply
113155		8375		135786	
		25125		67893	
		259625		14716	the two chances
		113155	Added	372780	Deducted from chances of A
		372780	Of chances of A	441936	for the chance of B

Both part by 36. There comes 10355 for the chance of A, 12276 for the chance of B. That which it was necessary to demonstrate.