ÉCLAIRCISSEMENT relatif au Mmoire sur la mortalit de la petite vrole qui se trouve dans le Volume de 1796.*

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Mémoires de l'Académie Royale des Sciences et Belles-Lettres... Berlin
1804, Class de math., pp. 80–82

§ 15 of this Memoir contains many faults of impression, which it is important to observe. At line 13, instead of *being divided by* 42, put *being added to* 1300, *then divided by* 2.42. At line 22, instead of *divided by* 36, put *added to* 887, then *divided by* 2.36. At line 30, instead of *divided by* 34.7, put *added to* 643, then *divided by* $2.34, 7$. At line 39, instead of *divided by* 30.4, put *added to* 506, then *divided by* $2.30, 4$. At line 48, instead of *divided by* 44, put *added to* 403, then *divided by* 2.44. (I count the lines from the beginning of the paragraph.)

Of the rest, I must caution that the method of approximation that I have given in this memoir as a test, by awaiting that some more detailed observations put us in a state to proceed with more regularity, that this method, I say, is worth absolutely nothing, & I owe some excuses to the public in order to have presented it to them. Although this approximation satisfies rather well to the observations, however, if one proceeds rigorously, one finds some essentially different results. Now, without leaving some assumptions that I have employed in this approximation, one can proceed rigorously as one is going to see. I resume in § 11 the formula $b^{(i)} =$

$$\frac{a^{(i)}b^{(i-1)}\left(1-\frac{1}{n^{(i-1)}}\right)}{a^{(i-1)}-\frac{b^{(i-1)}}{m^{(i-1)}n^{(i-1)}}}.$$

Now, let $g^{(i-1)}$ be the number of deaths of smallpox, the number of those who would have contracted smallpox will be $m^{(i-1)}g^{(i-1)}$; subtracting this number from $b^{(i-1)}$, one will have $b^{(i-1)} - m^{(i-1)}g^{(i-1)}$ for the number of persons exempt from smallpox reported at the beginning of the year. For it to report at the end, following the assumptions of the approximation, it is necessary to multiply by $\frac{a^{(i)}}{a^{(i-1)}}$, this which gives a second value of $b^{(i)}$. Matching these two values, one has by dividing by

$$a^{(i)}\frac{a^{(i)}b^{(i-1)}\left(1-\frac{1}{n^{(i-1)}}\right)}{a^{(i-1)}-\frac{b^{(i-1)}}{m^{(i-1)}n^{(i-1)}}} = \frac{b^{(i-1)}}{a^{(i-1)}} - \frac{m^{(i-1)}g^{(i-1)}}{a^{(i-1)}},$$

*Translated by Richard J. Pulskamp, Department of Mathematics & Computer Science, Xavier University, Cincinnati, OH. December 22, 2009 whence one deduces

$$m^{(i-1)}n^{(i-1)} = \frac{a^{(i-1)}b^{(i-1)}m^{(i-1)} + b^{(i-1)}m^{(i-1)}g^{(i-1)} - b^{(i-1)2}}{a^{(i-1)}m^{(i-1)}g^{(i-1)}}.$$

Now

$$b^{(i)} = \frac{a^{(i)}b^{(i-1)} + a^{(i)}m^{(i-1)}g^{(i-1)}}{a^{(i-1)}},$$

$$\frac{b^{(i)} + b^{(i-1)}}{2g^{(i-1)}} = m^{(i-1)}n^{(i-1)} = \frac{a^{(i)}b^{(i-1)} + a^{(i-1)}b^{(i-1)} - a^{(i-1)}m^{(i-1)}g^{(i-1)}}{2a^{(i-1)}g^{(i-1)}}$$

Matching these two values of $m^{(i-1)}n^{(i-1)}$ & resolving the equation of the second degree, one finds

$$m^{(i-1)} = \frac{b^{(i-1)}}{a^{(i)}g^{(i-1)}}\left(\frac{(a^{(i-1)} - a^{(i)}}{2} + g^{(i-1)}\right)\left\{\sqrt{1 + \frac{2a^{(i)}g^{(i-1)}}{\left(\frac{a^{(i-1)}-a^{(i)}}{2} + g^{(i-1)}\right)^2}} - 1\right\},$$

$$b^{(i)} = \frac{b^{(i-1)}}{a^{(i-1)}}\left\{\frac{a^{(i-1)} + a^{(i)}}{2} + g^{(i-1)} - \left(\frac{a^{(i-1)} - a^{(i)}}{2} + g^{(i-1)}\right)\sqrt{1 + \frac{2a^{(i)}g^{(i-1)}}{\left(\frac{a^{(i-1)}-a^{(i)}}{2} + g^{(i-1)}\right)^2}}\right\}.$$

Calculating according to these formulas the observations reported in the memoir cited, I have found that the values of n, instead of remaining near the same, vary enormously and more than the values of m. I have prepared some tables of these variations, but I have suppressed them because I have made reflection that all this calculation rested at the 'bottom under some more or less arbitrary assumptions, & that the slightest change in these assumptions gave of all other results. If one makes

$$m^{(i-1)}n^{(i-1)} = \frac{b^{(i-1)}}{g^{(i-1)}},$$

$$\& \, b^{(i-1)} = \frac{a^{(i)}}{a^{(i-1)} - g^{(i-1)}}(b^{(i-1)} - m^{(i-1)}g^{(i-1)}),$$

the two formulas are reduced to one, & the calculation falls.