DÉMONSTRATION Mathématique du soin que Dieu prend de diriger ce qui se passe dans ce monde, tirée du nombre des Garçons & des Filles qui naissent journellement*

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The contemplation of this Universe, & of that which we see to happen every day, furnishes us an infinity of proofs of the existence of a Being, who not only has created the heavens & the earth, & has regulated the course of it by subjecting them to some fixed & immutable laws, but who still directs continually that which happens. The creatures who appear to us to be of the least consequence, the events which seem hardly to merit our attention, would be able to furnish some capable reasons to close the mouth on the most subtle Atheists, & to demonstrate the existence of a God, if one proposed a way to make them sense all the force of it. 1.

The number of Infants who are born is an example of it; few people reflect on that which it offers of the remarkable to us; & which consists in this that there are born very nearly as many boys as girls, but in a way however the number of the former surpasses always slightly the number of the latter. This sole fact, examined with attention, proves demonstratively that the birth of Infants is directed by an intelligent Being, on whom it depends.

I am going to work to put this proof in all its days. For this I will limit myself to the Infants born in the City of London, & those solely during the period of eight-two years, namely from the beginning of 1629 to the end of 1710. I will give here the list drawn from the registers of the Infants who one baptizes there: registers which one conserves in the Churches of that City; & I will leave to the Reader to judge of the new degree of force which this proof would acquire, if one would apply the calculus which I am going to make to each country, & to a longer sequence of years. 2.

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LIST

Of the male & female Infants who have been baptized in London during 82 years. 3.

Years	Boys	Girls	Years	Boys	Girls
1629	5218	4683	1670	6278	5719
30	4858	4457	71	6449	6061
31	4422	4102	72	6443	6120
32	4994	4590	73	6073	5822
33	5158	4839	74	6113	5738
34	5035	4820	75	6058	5717
35	5106	4928	76	6552	5847
36	4917	4605	77	6423	6203
37	4703	4457	78	6568	6033
38	5359	4952	79	6247	6041
39	5366	4784	80	6548	6299
40	5518	5332	81	6822	6533
41	5470	5200	82	6909	6744
42	5460	4910	83	7577	7158
43	4793	4617	84	7575	7127
44	4107	3997	85	7484	7246
45	4047	3919	86	7575	7119
46	3768	3395	87	7737	7214
47	3796	3536	88	7487	7101
48	3363	3181	89	7604	7167
49	3079	2746	90	7909	7302
50	2890	2722	91	7662	7392
51	3231	2840	92	7602	7316
52	3220	2908	93	7676	7483
53	3156	2959	94	6985	6647
54	3441	3179	95	7263	6713
55	3655	3349	96	7632	7229
56	3668	3382	97	8062	7767
57	3396	3289	98	8426	7626
58	3157	3013	99	7911	7452
59	3209	2781	1700	7578	7061
60	3724	3247	1	8102	7514
61	4748	4107	2	8031	7656
62	5216	4803	3	7765	7683
63	5411	4881	4	6113	5738
64	6041	5681	5	8366	7779
65	5114	4858	6	7952	7417
66	4678	4319	7	8379	7687
67	5616	5322	8	8239	7623
68	6073	5560	9	7840	7380
69	6506	5829	10	7640	7288

One sees in this List. 1°. That the number of boys has always surpassed that of the 4.

girls.

2°. That the difference between these two numbers has always been of a certain quantity, except that always the number of boys has passed it, in order to approach more nearly the one of the girls.

3°. That the difference between these two numbers is always remained between certain limits, slightly extended the one from the other.

If these births had depended upon chance, none of these things would have happened: because as then there would have been a like probability for the birth of a boy as for that of a girl, there would have resulted from it as often the number of girls would have surpassed the one of the boys; as often also these two numbers would have been equal, very little short of; and that sometimes also they would have differed by much. 5.

But in order to render the thing more sensible, I am going to determine rightly how much it would be to wager against one, that this which is arrived in London during 82 years, would not be arrived by supposing that the birth of the infants is the effect of chance. That which I will say over there, will convince all those who make some usage of their reason, that this birth arrives in consequence of a particular direction from Providence.

Before coming to the calculus necessary for this, it is à propos to make some remarks on the manner in which it is necessary to take in order to determine the chances in games, or the other things which depend on chance, & to give some general rules, so that those who are a little exercised in the elements of Algebra, but who have never thought on this matter, are set thence in a state to follow my calculation & to understand my demonstration. 6.

For this I begin by remarking that in order to demonstrate the chances in question, it is necessary to seek the number of all the cases which are able to arrive with the same facility, & which bring either gain or loss. This number one time found, it is necessary to distinguish the cases which are winning from with those which are losing, & then to find the value of chance, by paying attention to this; it is that the number of all the possible cases, is to the number of those which are able to make winning, as the amount of the wager is to the value sought. 7.

Suppose for example, that among 20 equally possible cases, there are 15 of them which are able to make me win the sum A; & only 5 which are able to make me lose; then I say as 20 is to 15, thus A is to the value of my chance, which consequently is $\frac{3}{4}$ of A. Thus there are odds of 3 against 1, in my favor, because there are 3 cases which are able to make me win, & only one which is able to make me lose. This is so clear, that it is not necessary to be stopped longer to demonstrate it.

I will add only here an example, so as to make better understand my thought to those who are not accustomed to regard these things with a mathematical eye. If someone casts a die, there are six cases which are able to arrive with the same facility. If I wish to wager that on the first cast he will bring forth six points, it is evident that his lot is worth only $\frac{1}{6}$ of the price on which one is agreed: thus for the equality of things, the one with whom he wagers must set 5 against 1. This which accords with that which I just said.

I will serve myself by a similar reasoning in order to clarify the question on the birth of the Infants: but before I will demonstrate some propositions, which will serve to find the number of all the cases which are able to arrive, & to distinguish those from among these cases which give that which is arrived, from those which would give the contrary.

These propositions will turn on some tokens, which cast down at random fall heads or tails: the uncertainty of the side, which they will offer in falling, is very well able to be compared with that which there is if an Infant will be born male or female. 8.

FIRST PROPOSITION

If one casts into the air a determined number of tokens, they are able to fall in many different ways; this which gives a certain number of cases which are able to arrive. But if one increases by one the number of tokens, I say that then the number of cases which are able to arrive is double of that which was before this addition. 9.

DEMONSTRATION

All the cases which are able to arrive with the first tokens, are equally possible, if the one which one has added falls heads: they are still equally possible, if it falls tails. Therefore the number of possible cases is double. That which it was necessary to demonstrate.

FIRST COROLLARY

It follows from this proposition, that a token being able to give two cases; two tokens will give four of them, three will give eight of them, four will give sixteen of them, & thus consecutively. Consequently, the number of all the cases which are able to arrive, when one casts a determined number of tokens, is able to be expressed by the number 2 carried to the power of which the exponent is the same as the tokens. For example, if the number of tokens is n, the number of all the cases will be 2^n. 10.

SECOND COROLLARY

That which I just said (9) of the number of cases which becomes double by the addition of one token, & of one token (10) which gives only two different cases, is able to be applied to all sorts of numbers, variables according to the circumstances. It is thus, for example, that if one adds one die to a determined number of other dice, the number of the possible cases become sextuple of that which it was before, & that because the added die is able to fall in six different ways, of which each is able to unite with all the cases, that the dice taken firstly are able to give: thence it follows that if n designates the number of dice, 6^n will designate the number of all the possible cases; & it is likewise of each other number of them. 11.

SECOND PROPOSITION

If k expresses heads, & m tails, $1k+1m$ or $k+m$ will express the number of cases which a token is able to give. Consequently if one casts into the air a given number of tokens, one will find the number of all the cases, that they will be able to give (10), in raising $k+m$ to the power of which the exponent is the number of tokens. This being, I say that the number of cases, in which one part of the tokens fall heads, while the other 12.

fall tails, will be expressed by the coefficient of the term in which the exponent of k is the same number as the one of the cases which must give heads.

For example: if there are 5 tokens, the number of all the possible cases is

$$k^5 + 5k^4m + 10k^3m^2 + 10k^2m^3 + 5km^4 + k^5.$$

Now among all these cases, how many of them are there which will give 3 heads & 2 tails? I say that there are 10, since the coefficient of k^3m^2 is 10.

DEMONSTRATION

$k + m$ expresses the two cases which are able to arrive with one token; if I employ a second of it, I must multiply $k + m$ by k, because the preceding cases are able to arrive when this second token falls heads: it is necessary also that I multiply $k + m$ by m, because these same cases are able again to arrive, when the second token falls tails. Thus the multiplications by k express the tokens which fall heads, & those, which are made by m indicate the pieces which fall tails: consequently the number of each letter in the product expresses the number of tokens which are able to fall heads or tails. Thence it follows, that among

$$k^5 + 5k^4m + 10k^3m^2 + 10k^2m^3 + 5km^4 + k^5,$$

which make the 32 possible cases with 5 tokens, there are 10 cases where three tokens fall heads, & two fall tails, because one finds $10k^3m^2$. The same demonstration shows us further that there are also 10 cases, in which two tokens fall heads & three tails, that there are five cases where four tokens give heads, & one tail, & five other cases where four tokens offer tails, & one heads; finally that there is only one case alone where all the tokens fall heads, & one also where one has five tails. That which one just said of five tokens, is able to be applied to any other number.

COROLLARY

This proposition gives us the means to find the lot of a player, who would wager 13.
that among a determined number of tokens, the number of those which would fall heads will be between two given limits.

A, for example, *has wagered against B, that among nine tokens the number of those which would fall heads will be between 2 and 6, that is to say that it will not be greater than 5, nor less than 3. One demands the value of the lot of A.*

RESPONSE. All possible cases with 9 tokens are

$$k^9 + 9k^8m + 36k^7m^2 + 84k^6m^3 + 126k^5m^4 + 126k^4m^5 + 84k^3m^6 + 36k^2m^7 + 9km^8 + m^9,$$

which are together 512 cases. Among these cases, according to that which has been said above (12.), there are 84 of them which give 3 heads & 6 tails; there are 126 of them which give 4 heads & 5 tails, & as many which give 5 heads & 4 tails. Thus among the 512 possible cases there are 336 which are to the advantage of A; the others make a loss to him. Consequently, the value of its lot (7.) is $\frac{336}{512}$ of the price of the wager; & since B has for himself all the cases which are disadvantageous to A, his lot is expressed by $\frac{176}{512}$. One sees thence that the lot of A is to the one of B as 336 is to 176, that is to say as $1\frac{10}{11}$ is to 1. Thus the lot of A, is worth nearly the double of the one of B.

It is therefore easy, as it appears, to resolve similar questions, when the numbers are small; but if they are quite great, it would be too inconvenient to use this method; it is necessary then to seek an abridgment: this is to what the following considerations are able to serve.

In order to respond to these sorts of questions, it is not always necessary to make use of the coefficients which we just employed, one is able to substitute other numbers which have among them the same proportion. In the preceding question, for example, instead of the numbers 1, 9, 36, 84, & 126, one would have been able to use their halves, their fourths, &c. or some numbers double, triple, quadruple, &c. unless that had given some changeover in the value of the lots. If one had made use of the halves, one would have found that the value of the lot of A is $\frac{168}{256}$ instead of $\frac{336}{512}$, now these two quantities are equals. This is too clear for the sake that one must give a more ample demonstration. 14.

In order to draw part of this remark, it is necessary to examine the formation of the coefficients that one obtains in raising a binomial to any power. Let be given $k+m$ to raise to the power of which the exponent is n. 15.

It is manifest that I am able to assume that I have as many of the different quantities k & m, as there are units in n, & that all these quantities, multiplied the ones by the others, must give the product that I seek.

It is clear also that in this product, I will have as many of $k^{n-1}m$, as one is able to have by the multiplication of all the different quantities $k+m$. This is able to be said also of $k^{n-2}m^2$, & thus consecutively of all the other products which are together $\overline{k+m}^n$. I suppose therefore that all the diverse quantities $k+m$, are expressed by K+M, k+m, $\mathfrak{K}+\mathfrak{M}$ &c. The first member of the sought power will be the product of all the different k, or k^n =Kk$\mathfrak{K}$ &c., up to the number of n. In order to find the coefficient of the second member, that is to say, how many times one will have $k^{n-1}m$, one must remark that in the first member Kk$\mathfrak{K}$ &c., instead of K one is able to put M; instead of k, m; instead of $\mathfrak{K}$, $\mathfrak{M}$, & thus consecutively, until one has changed each k one time; that which shows that the sought coefficient is the number of all the k, that is to say n.

If one goes further, & if one changes in each $k^{n-1}m$ all the k the one after the other, the number of all the $k^{n-2}m^2$ will be $n-1$ times as great as the number $k^{n-1}m$, which to it likewise is n: & consequently one will have $\frac{n}{1}\times\overline{n-1}\,k^{n-2}m^2$. But if in k^{n-1}M, one has substituted m instead of k, one has obtained k^{n-1}Mm, & if in $k^{n-1}m$, instead of k one has put M, one has found the same quantity k^{n-1}Mm. Now one is able to say the same thing of all the others $k^{n-1}m$, where one brings always an m: thus $\frac{n}{1}\times\overline{n-1}\,k^{n-1}m^2$ is composed of quantities which are taken each twice, & that it is necessary consequently to divide by 2, in order to have the third member, which will be $\frac{n}{1}\times\frac{n-1}{2}\,k^{n-2}m^2$. 16.

In order to find the coefficient of the fourth member, one must change all the k in particular in each $k^{n-2}m^2$, & by this operation one will have $\frac{n}{1}\times\frac{n-1}{2}\times\overline{n-2}\,k^{n-3}m^3$. But here it is necessary to note that if in $k^{n-2}M$m, one brings $\mathfrak{M}$, & if in $k^{n-2}M\mathfrak{M}$, 17.

one brings m, & finally if in $k^{n-2}\mathrm{m}\mathfrak{M}$, one brings M, one has each time the same product $k^{n-3}M\mathrm{m}\mathfrak{M}$, that which shows that $\frac{n}{1} \times \frac{n-1}{2} \times n-2\,k^{n-3}m^3$ is composed of products of which each is taken three times, & is consequently three times too great; whence it follows that it is necessary to make the division by 3 this which gives, $\frac{n}{1} \times \frac{n-1}{2} \times \frac{n-3}{3}\,k^{n-3}m^3$.

One will prove in the same manner that the coefficient of the fifth member is $\frac{n}{1} \times \frac{n-1}{2} \times \frac{n-2}{3} \times \frac{n-3}{4}$, & that the one of the sixth is $\frac{n}{1} \times \frac{n-1}{2} \times \frac{n-2}{3} \times \frac{n-3}{4} \times \frac{n-4}{5}$, & thus consecutively for all the coefficients, that one will have found all when this operation will have been reiterated as many times as there are units in n. 18.

From this that I just said one is able to deduce the following Corollaries.

FIRST COROLLARY

In the preceding numbers, which are multiplied by one another, one sees that those above them always diminish, while those below them increase: whence it follows that all the numbers, in which n is diminished, by subtraction, to the half, are some fractions less than unity, & that consequently make the quantities diminish that they multiply. Thus the coefficients, which always go by increasing, to that which n is diminished to the half, thence go always by diminishing, & this in a way that all the coefficients already found reappear again: consequently one half of the coefficients being found, the other half is also. One will understand better the thing & its reason, if one wishes rightly to take the pain to take instead of n any number whatever, & to form the coefficients from it according to the method which we just indicated. 19.

SECOND COROLLARY

That which I have said on the formation of the coefficients furnishes the means to find the coefficient which precedes, or the one which follows immediately a given coefficient. Let, for example, c be the coefficient of the tenth member of $\overline{k+m}^n$; one will have the coefficient of the eleventh, if one multiplies c by $\frac{n-9}{10}$, & the one of the ninth by dividing c by $\frac{n-8}{9}$. 20.

We pass, at present to the question which we ourselves have proposed to solve, by the remarks which we just made. 21.

The question is to find how much there would be to wager against one, that that which is arrived at London would arrive not arrive at all, if the birth of the Infants depended on chance.

In order to render the calculus easier, I consider that instead of the Infants born each year, & of which the number has continually varied, as one sees it in the list that I have given (3.), I would be able to use a mean number, by supposing that each year, there had been the same number, of Infants born, but in a way that each year there has been among the number of boys & that of the girls, the same proportion who are found in the numbers of the Table (3.)

In order to find this mean number, I put in a sum the numbers of all the Infants born in London during 82 years, as the Table indicates them: I take the eighty-second part of this sum, & I find that since the beginning of 1629 to the end of 1710, there is born each year in London, taking one year with the other, 11429 Infants, as many males as females. 22.

In the year 1703 the number of boys is the nearest to that of the females. There is born, this year here, 7765 boys, & 7683 girls, who are altogether 15448 Infants. Instead of this last number, if one take the mean number 11429, one will have for this year 5745 boys, & 5684 girls; so that the number of boys who are born, has surpassed by 30 the half of the one of all the Infants. 23.

In 1661, the excess of the number of boys over the one of the girls has been the greatest; & the difference of these two numbers being carried on the number 11429, one finds 6128 boys, & 5301 girls: consequently the number of boys has surpassed by 423 the half of the number of all the Infants born this year. Thus in supposing that there is born annually 11429 Infants, the number of boys has never been under 5745, nor above 6128, so that the greatest difference that there has been between these numbers, during 82 consecutive years, has been 383.

I am now going to the calculus by which I must resolve the question, & in order to render it more simply I demand first. 24.

If one casts at the same time 11429 tokens, what is the lot of A, who has wagered against B, that the number of those which will fall heads will not be less than 5745 nor more than 6128?

Or, that which reverts to the same,

What is the lot of A, who has wagered against B, that of 11429 Infants, who will be born in a year, the number of males will be contained between the two limits of the numbers 5745 & 6128?

This question is manifestly the same as that which has been made before No. 13: there is difference only in the magnitude of the numbers. In order to respond according to the method which has been followed in the same No. 13 by supposing that g designates a boy, & f a girl, it will be necessary to raise the binomial $g + f$ to the power of which 11429 is the exponent, & it will be necessary to put into a sum the coefficient of $g^{6128} f^{5301}$, & the one of $g^{5745} f^{5684}$, & all the intermediaries. Thence one will find the lot of A is to that of B, as the sum of all these coefficients is to the sum of all the others. But as these coefficients are of numbers too great in order to be handled easily, it will be necessary to employ some other numbers, which are proportionals to them. In order to find them, one must note that $g^{5715} f^{5714}$ & $g^{5714} f^{5715}$, have the same coefficient, which is at the same time the greatest: I call this coefficient c. In order to find the coefficient which immediately precedes it, that is to say the coefficient of $g^{5716} f^{5715}$, which is the 5714th member of $\overline{g+f}^{11429}$, it is necessary to divide the coefficient c (20.) by $\frac{11429-5713}{5714} = \frac{5716}{5714}$, this which will give $\frac{5714}{5716}c$. In order to find next the coefficient which precedes this one, it will be necessary to divide by $\frac{11429-5712}{5713} = \frac{5717}{5713}$, this which gives $\frac{5713\times5714}{5717\times5716}c$, which is the coefficient of $g^{5717} f^{5712}$. In the same manner one will find that $\frac{5712\times5713\times5714}{5718\times5717\times5716}c$ is the coefficient of $f^{5718} g^{5711}$, & in this manner one will be able to find all the other coefficients. Now as all these coefficients thus found, are multiplied by c, it is clear that by taking in the place of c any number at will, one will have, in the place of the found coefficients, some other numbers which will be proportional to them, & of which one will be able to make use (14.)

I suppose therefore that c, instead of designating the coefficient of $g^{5715} f^{5714}$, is equal to 100000, that is to say, that it is 100000 cases in which 5715 boys are able to be born. The number of cases in which 5716 boys are able to be born is (24.) 25.

$\frac{5714}{5716}c.$ Instead of c putting 100000, one will have $\frac{5714}{5716}\,100000 = 99965$. One will find in the same manner that the number of cases in which 5717 boys will be able to be born, will be $\frac{5713}{5717}\,99965 = 99895$, & thus consecutively, as one will see in the following Table.

TABLE 26.

Of the cases or of the chances for the number of boys, who are born among 11429 Infants. The number of the chances for the birth of 5715 boys, being supposed 100000.

Number of boys	*Number of chances*	*Number of boys*	*Number of chances*
5715	100000	5734	93546
5716	99965	5735	92893
5717	99895	5736	92213
5718	99790	5737	91506
5719	99651	5738	90772
5720	99454	5739	90013
5721	99245	5740	89229
5722	99002	5741	88421
5723	98725	5742	87589
5724	98415	5743	86735
5725	98071	5744	85859
5726	97693	5745	84962
5727	97285	5746	84064
5728	96843	5747	83110
5729	96370	5748	82115
5730	95865	5749	81184
5731	95330	5750	80195
5732	94765	5751	79191
5733	94170	5752	78173
5753	77140	5797	30387
5754	76094	5798	29516
5755	75037	5799	28661
5756	73967	5800	27821
5757	72888	5801	26996
5758	71800	5802	26187
5759	70702	5803	25393
5760	69598	5804	24614
5761	68486	5805	23851
5762	67369	5806	23103
5763	66247	5807	22371
5764	65120	5808	21654
5765	63991	5809	20954
5766	62859	5810	20268

Number of boys	*Number of chances*	*Number of boys*	*Number of chances*
5767	61725	5811	19597
5768	60591	5812	18960
5769	59457	5813	18306
5770	58323	5814	17682
5771	57191	5815	17074
5772	56062	5816	16481
5773	54935	5817	15903
5774	53813	5818	15340
5775	52694	5819	14792
5776	51581	5820	14258
5777	50474	5821	13739
5778	49373	5822	13234
5779	48280	5823	12743
5780	47194	5824	12266
5781	46116	5825	11801
5782	45048	5826	11353
5783	43988	5827	10961
5784	42939	5828	10493
5785	41900	5829	10083
5786	40871	5830	9685
5787	39854	5831	9300
5788	38849	5832	8926
5789	37856	5833	8565
5790	36875	5834	8216
5791	35907	5835	7878
5792	34952	5836	7551
5793	34011	5837	7236
5794	33084	5838	6931
5795	32170	5839	6636
5796	31271	5840	6352
5841	6078	5885	617
5842	5814	5886	581
5843	5559	5887	548
5844	5314	5888	515
5845	5077	5889	485
5846	4850	5890	456
5847	4631	5891	429
5848	4420	5892	403
5849	4218	5893	379
5850	4023	5894	356
5851	3836	5895	334
5851	3656	5896	313
5853	3484	5897	294
5854	3319	5898	278

Number of boys	*Number of chances*	*Number of boys*	*Number of chances*
5855	3160	5899	259
5856	3008	5900	242
5857	2862	5901	272
5858	2722	5902	213
5859	2588	5903	199
5860	2460	5904	186
5861	2338	5905	174
5862	2220	5906	163
5863	2108	5907	153
5864	2001	5908	143
5865	1899	5909	133
5866	1801	5910	124
5867	1708	5911	116
5868	1619	5912	108
5869	1534	5913	101
5870	1453	5914	94
5871	1376	5915	88
5872	1302	5916	82
5873	1232	5917	76
5874	1165	5918	71
5875	1102	5919	66
5976	1041	5920	61
5977	984	5921	57
5978	930	5922	53
5979	878	5923	50
5980	828	5924	46
5981	782	5925	43
5982	737	5926	40
5983	695	5927	37
5984	655	5928	34
5929	31	5952	5
5930	29	5953	5
5931	27	5954	4
5932	25	5955	4
5933	24	5956	4
5934	22	5957	3
5935	20	5958	3
5936	19	5959	3
5937	17	5960	3
5938	16	5961	2
5939	15	5962	2
5940	14	5963	2
5941	13	5964	2
5942	12	5965	2

Number of boys	*Number of chances*	*Number of boys*	*Number of chances*
5943	11	5966	2
5944	10	5967	2
5945	9	5968	1
5946	8	5969	1
5947	8	5970	1
5948	7	5971	1
5949	7	5972	1
5950	6	5973	1
5951	6		

In order to avoid in this Table the fractions, one has neglected those which are less than $\frac{1}{2}$, but one has taken for 1 those which are greater. Thus the errors occasioned thence complement one another. I must again caution that this Table has been calculated by means of Logarithms, so that the fractions which have been increased, or those which have been disregarded, have not been able to cause the least change or the least error in the other numbers.

I remark again that this Table has not been pushed further, because the numbers, which express the following chances, are so small, that, compared with the others, they are not able to enter into consideration.

In order to continue to present our researches, it is necessary to observe that the 27. number of chances or cases which are in the Table for 5745 Infants, with all the numbers of the cases which follow, & which taken together are the sum of 3849092; that these numbers, I say, express the cases which make A win. Beyond this, A has again in his favor all the small fractions which would be coming, if one had continued the Table to the number of 6128. These fractions would not rise together to 50; but in order to take all to the greatest advantage of A, I suppose that they make 58; whence it follows that the number of all the cases which make A win is 3849150. The number of all the cases which are able to arrive is the double of the sum of all the Table, that is to say, the double of 6590400, this which makes 13196800, without including the sum of all the small numbers which one would have obtained, if one had pushed the Table further, this which causes a small error, but which is to the advantage of A, of which the lot is to the one of B as 3849150 to 9347650, which is the number of all the other cases; that is to say as 1 to $2\frac{32987}{76983}$, or as 1 to a little more than $2\frac{2}{3}$. Thus one is able to wager very nearly 3 against 1, that this which is arrived in London during 82 consecutive years, will not arrive in a determined year.

I go further: I suppose that A has wagered against B, that this same event will arrive 28. each year & that during 82 consecutive years. In order to find in this case the chance of A, I call a his lot, which comes to be found (27.) for a year, & I name b the lot of B. I raise $a+b$ to the 82$^{\text{nd}}$ power, this which gives me $\overline{a+b}^{82}$; by what all these cases are expressed (11.). Among all these cases, the only one which is favorable to A, is a^{82}, of which the coefficient is 1 (15.). Now a being taken equal to 1, A will have only one case in his favor, & all the others will be for B. But since $a=1$, one will have $b=2\frac{32987}{76983}$ (27.), & consequently $a+b=3\frac{32987}{76983}$, this which being carried to the eighty-second power, gives

$$\overline{a+b}^{82} = 75,598,215,229,552,469,135,802,469,135,802,469,135,802,469.$$

Now, I repeat it, in this immense number of cases, there is only a single one which is able to make A win; & however we have calculated his chance broadly enough (27.).

We see at present what are the consequences which result from all that which we just said. So that the best, & at the same time the most useful for the propagation of the human race, takes place, it is necessary that the number of men be very nearly equal to that of the women. But on the other hand, men being exposed to more perils than women, the number of those who perish that way, is greater than the one of women who die by the maladies peculiar to their sex. That way it follows naturally, that the conservation of the better order requires that, among the Infants who are born, the number of boys surpasses the one of the girls. But there are odds 29.

$$75,598,215,229,552,469,135,802,469,135,802,469,135,802,469 \text{ against } 1,$$

that in a city as London it will not happen during 82 consecutive years that the number of boys surpasses, at least by 60, the one of the girls. However this is arrived. Who would be therefore to disregard here the direction of Providence, who presides at the birth of the Infants?

If a man took at the edge of the sea a grain of sand, which was unique which was able to be useful to him, would one say that it is without choice that he has raised this grain of sand, & that it is by chance that it is found under his hand? But what is the number of grains of sand, in comparison to the number that we have found? If the entire globe of the earth, including the extent of the seas, were all formed of sand, the number of grains which would compose it, would not be yet the millionth part of our number.

The more one will reflect with attention on this, the more one will be struck in admiration. If one casts the eyes on the beginning of the Table which one finds above (26), one will see that the numbers of boys, contained between 5715 & 5745, & which designate the cases which are not at all arrived; one will see, I say, that these numbers, considered each separately, have much more probability, or a greater number of chances, than any of the numbers which express the one of the boys born in one of the 82 years of which there is question.

What concludes from all that which one just said? It is that the one who has created the Heavens & the Earth, directs that which passes not only by the general laws which he has established since the beginning of their existence, but further by the particular laws of which the effect makes itself felt every day. There is only one intelligent Being who is able to make some boys & some girls be born precisely as much as it is necessary of one another, in order that all remains in order, despite the prodigious probability that opposes it, if one pays attention only to that which is able to result from general & physical laws.

If one examined in the same manner all that which arrives on our globe, one would be convinced that not only there is nothing which escapes the knowledge of God; but that still some things, which seem to depend on these general laws, are directed always in a particular fashion by this supreme Being, to the conservation & to the benefit of his creatures.