INTRODUCTION

A LA

PHILOSOPHIE,

CONTENTANT

LA METAPHYSIQUE,

ET

LA LOGIQUE*

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An Extract from
Book II. *Logic*
Part I. *Concerning Ideas & Judgments*
pages 82–97

Chapter XVII. *Concerning Probability*

We have seen, that there is a complete difference between mathematical evidence, & moral Evidence. The first is the mark of Truth, by itself (457, 458);[1] & the second, by the will of God (488, 491, 492),[2] that is to say, by institution. 582.

As each Evidence is supported on a solid foundation, the persuasion which is 583.

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[1]These are from Chapter XII, *Concerning Evidence*:

457. This Evidence (immediate perception) is the characteristic mark of the Truth, for the Ideas of all that which we perceive immediately. This is to say, that this Evidence suffices to convince us fully, that the Idea which we acquire, agrees with that which we perceive immediately.

458. Because the same thing always agrees with immediate perception, that we have of it. When I think, the thought is not distinct, in my Mind, from the perception that I have of it. The joy, in my Mind, & the perception that I have of it, are one & the same thing. Thus, this perception must give me the true Idea of this joy.

[2]Likewise from Chapter XII, *De l'Evidence*:

moral Evidence is also entire, as that which is based on mathematical Evidence: by where it appears, that this persuasion is different from Certitude, which one calls commonly moral; & by what one understands a great Probability.

584. Probability keeps a mean between ignorance, & knowledge in which there is lacking nothing, that is to say, which must produce an absolute persuasion.

585. Probability has no place at all in mathematical Evidence. When there is a concern of this Evidence, our knowledge increases when we acquire new ideas of things, of which we have an immediate perception (457.),[3] or in comparing together the ideas that we have already (460.)[4]

586. The knowledge of each idea, which we acquire in this manner, is perfect; & one would not know how to imagine some mean between ignorance, & a certain knowledge, when there is a concern of that which is offered immediately to
587. our Mind (461).[5] This reflection would not know how to be applied to moral Evidence. When there is a question to acquire the knowledge of things, which are beyond us, the concurrence of many circumstances is nearly always necessary; if, while a part of the circumstances is found there, the rest are lacking, the persuasion touching the correctness between the idea & the thing, to which one reports it, is imperfect. Thus, there is able to be different degrees in this persuasion; & these degrees are, what we call degrees of Probability, that the concern is to examine at present.

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488. Thus, Sense leads to the knowledge of the Truth, because God has wished it thus: & the persuasion of the conformity of Ideas, that we acquire through the Senses, with the things that they represent, is complete.

491. Whence we conclude, that God has wished that Testimony is also a mark of the Truth. He has further given to Men the faculty to determine the qualities that a Testimony must have, in order that one adds faith.

492. We have said finally, that Judgments, which have for foundation Analogy, leads us also to the knowledge of things. And the correctness of the conclusions, which we draw from Analogy, is deduced from the same principle, that is to say, from the will of God, of which providence has placed Man in some circumstances, which impose on him the necessity of a short & miserable life, if he refuses to attribute to things, that he has not examined, the properties that he has found in other similar things, in examining them.

[3] See note to (582.).

[4] From Chapter XII, *De l'Evidence*:

460. We deduce also from that which we just said, that each Judgment is true. Because we perceive immediately the relation which there is between the Ideas, which are present in our mind (453); & because of this, that perception represents to us the true relation, that there is among these Ideas; because, in the times that I perceive one such relation, its Idea would not be separated from those that I compare.

and also, from Chapter XI, *Concerning Fatality*:

453. We return all our Judgments to the same class; because they are nothing but the Ideas of relation, which we perceive immediately, when the Ideas, that we compare together, are present in our mind.

[5] From Chapter XII. *De l'Evidence*:

461. It is thence, that one is able to render reason, why Evidence draws our consent, in an irresistible manner.

If there is found in a subject some thing, that we imagine must be found there, we will name that an *Event*; because in considering the thing in itself, it would have to be able to be otherwise: & the Probability is also well able to have relation to future events, as to those which present or past. 588.

All that which is able to contribute to form a proof, but which alone nevertheless does not form one, furnishes a certain degree of Probability. If I seek in what house a certain Man is held, & if I discover the city where he is, I have already some thing, which is able to draw me to the knowledge of that which I wish to know; but this does not suffice. If one indicates to me the street, the Probability increases; & in case that I undertake to determine the house, where he is found, the risk to deceive me, however great as it is able to be, will be less, than if it was necessary to choose in every city. 589.

One is able to see, by that which we just said, that Probability does not regard the same things, but the knowledge that we have of them; & that one is able to consider as one quantity, which goes by increasing, from the smallest degree of knowledge, to entire persuasion. 590.

It is for this reason, that we imagine Certitude as a whole, divisible into as many parts as one will wish; & that, in order to determine the Probability, we must assign the ratio that there is between this whole, and the part, which expresses that which is known. 591.

We suppose, that a Man exits from a vessel, in which there are eighty-four *Hollanders*, twelve *Englishmen*, & four *Germans*. I do not know from what nation is the one which will come to exit from the vessel; but the risk to deceive me will be less, & consequently, the Probability greater, if I affirm, that it is a *Hollander*, than if I suppose it *German*, or *Englishman*. 592.

However, the assertion of the one who would say that it is a *German*, would not be destituted of all Probability. 593.

In the first case, the Probability would be to Certitude, as 21 to 25 & in the second, as 1 to 25. We will indicate, in the following, the fundamentals of this sort of calculus. 594.

One calls *Verisimilitude* the Probability, which surpasses half-certitude. In ordinary usage, one calls *probable* that which has Verisimilitude. This is why, it is not necessary to confound that which is probable with that which has only some Probability. 595.

The half-certitude forms the Doubt properly so-called, & is able to be envisaged as a kind of equilibrium. 596.

The degrees of Verisimilitude increase, from Doubt to Certitude. 597.

One calls *uncertain* that of which Probability is less than half-certitude, & it is manifest, that it must also have here different degrees. 598.

The proof of the Probability of that which we examine, belong to the matter of Probability. Because the first thing, which it signifies to us to determine, with respect to all that which we wish to know, is to know if it is possible. Thus, the simple Possibility forms the first degree of Probability; but the least of all, & smaller than every assignable degree. Here is why, in practice, the simple knowledge of the Possibility is confounded with ignorance; although properly speaking, Possibility differs from perfect ignorance, for which Possibility likewise 599.

is uncertain. It is thus, for example, that which that one tells of Specters & Sorcerers, must be regarded but as fabulous, when even one would prove the Possibility of them.

600. One deduces sometimes the Probability of the consideration of the same thing; & sometimes of the Probability of the argument, on which the assertion is founded.

601. In each case, one serves oneself with the same rules, in order to determine the Probability; which is either simple, or composite.

602. I will speak of simple Probability in this Chapter; & I will indicate in the following Chapter, to what it is necessary to pay attention, when the concern is to consider together many Probabilities.

603. In order to determine the Probability, which results from the nature of the thing, it is necessary to assign the possible cases, among which the *Event*, of the Probability of which there is concern, is necessarily found; & these cases, if they are composite, must be divided in a manner, that all are able to arrive with an equal facility. And the Probability will be to Certitude, as the number of cases, in each of which the proposed *Event* takes place, to the number of possible cases.

604. For example, someone seeks the degree of Probability that there is, that he has eight points with two dice: the possible cases with two dice are, that one will bring forth 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, or 12. But these eleven cases will not arrive with the same facility: seven points are able to be brought forth in six ways; & twelve, or two, in one alone.

605. By dividing these eleven cases, one discovers, that two dice are able to give thirty-six different outcomes, & that all are able to arrive with the same facility; because to each of the faces of one of the dice, some one of the six faces of the other is able to correspond.

606. Among all these outcomes, there are five of eight points; consequently, as 5 is to 36, thus the Probability that one seeks is to Certitude (603.); & this Probability is worth $\frac{5}{36}$ of Certitude.

607. When one of the two numbers, which express the ratio that there is between the Probability & Certitude, is lacking, the Probability would not be determined.

608. When, in order to discover these numbers, one pays attention only to the ideas of things, one is found often very troubled; especially in the affairs which are encountered in the ordinary course of life. One is served, on these occasions, by another method, in order to determine the Probability, namely, by examining the events themselves.

609. We suppose, that an urn contains some black tickets & some white tickets; one demands what Probability there is, that the first, that one will draw, will be black.

This Probability is to Certitude, as the number of black tickets in the urn is to the number of all the tickets (603.). But both of these numbers are unknown.

610. By neglecting consideration of these numbers, we discover the proportion that we seek, if many ballots have already been drawn before, either they had been rejected from the urn, or not; because the number of all the ballots drawn

is to the number of blacks, which is found among these ballots, as Certitude to the Probability, that we seek.

In truth, it is able to have a small error; but, if the number of ballots, which have been drawn, is great, it is not necessary to be troubled by this error in practice. 611.

Because one has demonstrated mathematically, that by increasing the number of observations, the danger of deceiving oneself becomes small, to the point of vanishing near to the end.

One is able to employ this method with success, in order to determine the Probability of the life of Men; & with the help of a table, formed on a great number of observations, one is able to resolve a great number of useful questions. 612.

If the observations have relation to some more determined cases, as to some particular malady, the conclusion will be more precise also; & one will be able to assign the magnitude of the danger to which the life is exposed, of the one who has the malady in question. 613.

The peril in navigations is able to be determined in the same manner, in order to fix the price of assurance. 614.

If, of a thousand vessels, which have undertaken the same voyage, itis ten of them perished, the assurance is worth the hundredth part of the value of that which one assures; a proportion, which it will be necessary to increase, or decrease, according to the quality of the vessel, in case that it is known; because we suppose, that the observations have had relation to some ordinary vessels. 615.

If the observations had been made in regard to a thousand vessels, perfectly similar to the one of which there is concern, one would fix more exactly the price of assurance; to which however it would be necessary to add some gain, in favor of the assurer, because the assurance is a kind of business.

This method to determine the Probability, by means of a certain number of observations, has passed into usage. But, as the greater part of the cases are not marked exactly, & as Men often neglect to consider distinctly the events, which do not have a particular relation with them, they determine the Probability by a grosser estimation: & one calls prudent those, who, by paying attention to that which must enter into the calculation, deviate less from the truth than the others in some estimations of this kind. 616.

And if one is not astonished, that we return to one determined proportion, not only the things which depend on one regular cause, but likewise those which are entirely contingent: because nothing is irregular, or fortuitous, if one considers the same things. We call irregular, that of which we do not perceive regularity, because of the concurrence of many different causes, & we give the name of fortune, to that of which we do not perceive the liaison of dependence, with a determined cause, although this liaison is very real (90.).[6] 617.

It happens very often, that the regularity, which, in considering a small 618.

[6]From Part I, Chapter VII *De la Cause, & de l'Effet*:

90. Thus, each Effect has a Cause, on which it depends necessarily; but this necessity is different, according as the difference of the subject.

number of effects, escaped us, is developed to our eyes, by increasing the number of effects that we make to enter into the examination (611.). On how many causes do not the end of the life of Man depend? However, in a number of thirty or forty thousand Men, the sequence of those who die is regular. And this same sequence, if there is question of Men taken by chance among all the inhabitants of a country, is not troubled by an epidemic malady; & when likewise it happens there some derangement, this derangement take place only during a small number of years: during all the others, the sequence continues, as if there had not been extraordinary mortality.

619. It is necessary to observe moreover, on the subject of Probability, that often one discovers that there is a Probability, without which one is able to determine it; that which happens when we see that the event of which there is concern is found necessarily among some others, which are able to happen with the same facility, but of which the number is unknown to us.

620. A thousand inhabitants of a city are perished, by an unexpected accident; the Probability that Pierre, who stays in that city, is of the number of dead, is to Certitude, as one thousand to the unknown number of inhabitants of the city.

621. If the event, of which there is concern, is found among the others, which we regard as being able to happen with the same facility, & if the number of them is infinite, there is Probability no longer.

But if that same is unknown to us, namely, if this number is infinite or not; we are ignorant if there is some Probability, or if there is not.

622. Those who claim, that there are some inhabitants in the planets, base themselves on the conformities that there is among the planets & our earth: conformities which prove the possibility of their assertion. But the Probability that the planets are inhabited, is to Certitude, as this particular usage of the planets, that is to say, as unity, is to the number of usages to which the planets are able to be destined. Now, who will dare to affirm, that this number is not infinite?

623. We have said, that Probability has some relation to the arguments, on which such or such assertion is based (600.). This sort of Probability is found in the same manner, as if it were a question of an event.

624. It is necessary to examine how many times, in a certain number of cases, where an argument has been employed, this argument has not deceived; & this first number will be to the number of cases, as the Probability of the argument to Certitude.

625. If I hear tell one hundred things by a Man, that he assures to have all views, & that he has said the truth ninety times; when he will tell some thing in a sequence, the Probability of his testimony will be worth $\frac{9}{10}$; & this Probability of the argument indicates the Probability of the thing, which is based only on this single argument.

626. Each Probability must be distinguished from Certitude; because, as we see sometimes to happen that which was scarcely probable, it is able to be also that a very probable event not happen.

627. However, Probability is able to be increased to the point of no longer being able to be distinguished from Certitude. Pierre seeks Paul, who is hidden. He

renders himself to the city where Paul is; he enters into a house, & goes straight ahead to the place, which hid the one who he seeks. I say, that this place has been known to Pierre: no person will disown it. However, there is, in favor of my assertion, only a very great Probability; because the contrary has some Probability, although very small; & this last Probability is to Certitude, as unity to the number of all the places where Paul has been able to be hidden.

Chapter XVIII. *Concerning Composite Probability*[7]

When many simple Probabilities must be considered together, we give to them the name of *composite Probability*, (601, 602.). The cases where the thing takes place, are rather frequent, & extremely varied. 628.

I. With respect to two or many events, of which the Probability is given, one demands what is the Probability, that in a determined case, the one or the other, or if it has many of them, one of all, happens. 629.

This question is resolved by the addition of all the given Probabilities; & the sum gives the Probability that one seeks. 630.

But it is able also to be resolved directly, as if it was a question of a simple Probability.

In the example of No. 592. if someone demands what is the Probability, that it is a Hollander, or a German, who is exited from the vessel, the sum of the Hollanders & the Germans, taken together, is to the number of all those who are in the vessel, that is to say, 88 is to 100, or 22 to 25, as the Probability is to Certitude (603.); this is why this Probability is worth $\frac{22}{25}$, which is the sum of the two separated Probabilities $\frac{21}{25}$, & $\frac{1}{25}$, (594, 603.). 631.

With two dice, one is able to bring forth eight points in five ways, & nine in four ways; thus the Probability that someone will bring forth *eight* or *nine*, is worth $\frac{9}{36}$; that is to say $\frac{1}{4}$ (603, 605, 606.).

II. The Probabilities of two events being given, one demands the Probability that there is, that the one or the other happen, in some distinct cases. One demands, for example, what is the Probability, that with two dice someone brings forth on the first trial *eight*, or on the second *nine*. 632.

The Example will be precisely alike, if Jean would be engaged to pay one thousand florins, in case that after ten years Pierre or Paul, of whom the age is given, is alive: one demands, what is the Probability, that Jean will pay the said sum? It is necessary to determine separately, in regard to Pierre & to Paul, what is the Probability that they will be alive after ten years; & next, it will be necessary to make a calculation, of which I will explain the method by resolving the question of No. 632. 633.

The Probability of the first event is worth $\frac{5}{36}$ (606.), & of the second $\frac{4}{36}$, or $\frac{1}{9}$; that which one finds in the same manner (603.). 634.

[7]Those who are not rather versed in Arithmetic are able to pass this Chapter.

The probability of the first outcome differs from Certitude, which is expressed by unity, by $\frac{31}{36}$; that is to say, that $\frac{31}{36}$ forms the contrary Probability.

This Probability $\frac{31}{36}$, from the first outcome, must be increased, & the contrary Probability diminished, because there remains a second trial to draw. The Probability of this second outcome is $\frac{1}{9}$; but if one would add this last Probability all entire, the first would be increased too much, the second trial not taking place at all, if one brings forth eight on the first trial. Thus the increase must not be worth a $\frac{1}{9}$ of entire Certitude, but only $\frac{1}{9}$ of the Probability that the second trial will take place; which is the same as the Probability of the contrary, in the first trial, & worth $\frac{31}{36}$: the ninth part of this fraction is worth $\frac{31}{324}$, which added to $\frac{5}{36}$ gives $\frac{19}{81}$; a number that we find with more ease by the following rule, that it is always necessary to employ in some similar occasions, & which extends itself to any number of Probabilities, that one considers at the same time.

635. We call *Complement of a Probability*, that which is lacking to this Probability, in order to attain unity, which expresses Certitude.

636. Rule, *By multiplying the complements of all the Probabilities*, either that there may be two of them, or further, *the complement of the product will give the Probability that one seeks.*

637. In the example of No. 633. we suppose, that Pierre has 28 years, & Paul 47. The Probability, that the first will be alive after ten years, is worth $\frac{17}{20}$; the Probability of the life of the second, after ten years, is worth $\frac{7}{9}$; the complements are $\frac{3}{20}$ & $\frac{2}{9}$, the product of these complements is $\frac{1}{30}$, of which the complement, namely $\frac{29}{30}$, expresses the Probability that one seeks; consequently, 29 is to 30; as the Probability that Jean will pay the sum of one thousand florins , is to Certitude; & this last has more promise, than if, without any condition, he would be engaged to pay in ten years nine hundred sixty florins.

638. The calculation must be made the same, if Pierre & Paul were of the same age, or if someone had undertaken to bring forth with two dice eight points, on the first or on the second trial: because, in this case, it would be necessary only to apply the rule to some equal Probabilities.

639. One thing, to which it is necessary to well take guard, is to not take for equals some Probabilities which are not such; this which is able to happen, when the union of two Probabilities bring some change to one of them.

640. Suppose, in the example of No. 592, that two Men exit from a vessel, & that one demands the Probability that there is, that one or the other is a Hollander. The Probability, in regard to the first, is worth $\frac{84}{100}$ (603.), that is to say $\frac{21}{25}$; in regard to the second, it is not the same; because, when the first is exited, there remains only 99 Men in the vessel; & at least if not all the Hollanders would be in this last number, it is not necessary that the second, who is exited, enters into consideration; consequently, the Probability is worth $\frac{84}{99}$ (603.). Thus, these are the Probabilities $\frac{21}{25}$ & $\frac{84}{99}$ which, according to the given rule (636.), must enter into the calculation.

641. III. Many questions, different from those which we have just proposed, are able to be brought back to these sorts of questions; in a manner that, in order to resolve them by the same rules, there is nothing necessary other than to change

the exposition of the case.

One demands, touching that which has been said, No. 633. & 637, what is the Probability that Pierre & Paul will die, both in the space of ten years? This question coincides with that same here that we have treated; & the Probability, that one seeks, is worth the Probability of the contrary of that which one would demand in number 637; & it is worth the difference that there is between the Probability, sought in this place, & Certitude. 642.

Similarly, if one demands what is the Probability, that Pierre & Paul will be both alive in ten years? This Probability is the same as that of the contrary, or of the lacking from Certitude of this other Proposition, that one or the other will die in the marked time; & this question belongs to those that we have explicated (632. & the following). 643.

But is is more commodious to resolve these sorts of questions directly; they belong to another class, of which we will treat in the following (651. & the following). 644.

IV. The Union of different probable arguments increases the Probability; however, the Probabilities of the arguments must not be joined together, by a simple addition. 645.

One proposes an argument, of which the Probability is worth $\frac{3}{4}$; consequently, that which remains of uncertainty is only $\frac{1}{4}$, & this is further diminished, if one adds a second probable argument: this second argument would not know how to take off all the uncertainty; this privilege appears only in a complete proof. If the Probability of the second argument is worth $\frac{2}{3}$, it will subtract two-thirds of uncertainty, that is to say, two-thirds of the fourth which remained, that is to say $\frac{1}{6}$ of all; by adding this $\frac{1}{6}$ to the Probability of the first argument, we will have the Probability that we seek, namely $\frac{11}{12}$. 646.

This reasoning returns to the one of No. 634. And the rule (636.), by which the question, proposed in this place, is resolved with ease, is able also to be applied here, when two, or a greater number of probable arguments, find themselves reunited. 647.

In order to apply this rule, it is necessary to examine, if the Probability of an argument is not changed by the concurrence of the other; this which happens, when the rule of No. 603. is able to be applied to the circumstances which contribute, considered conjointly. 648.

A Man has been killed by a sword strike, in a crowd: there is found that there has been only six Men who had drawn the sword, & Jacques has been one of the six; the Probability, that he is the author of the murder, is worth $\frac{1}{6}$ (603.). 649.

There is found, beyond that, that the one who has made the strike, had black hair; that in the crowd there had been ten, of whom the hair had this color, & that this second clue agrees again to Jacques. Thus, there is a new Probability, which is worth $\frac{1}{20}$, & which contribute with the first. The Probabilities, joined together, are worth $\frac{1}{4}$ (647, 636).

The Probability, that Jacques has committed the murder, is well determined in this manner here, if we know nothing further, than that which we just supposed. But, if we acquire some new light, the Probability is changed, that is 650 to say, increased, or diminished. If the two circumstances indicated would be able to be joined, & if there had been means to determine in how many Men these circumstances contribute, it would be necessary to consider only those Men alone. If the six Men, of whom first mention has been made, had black hair, the Probability would no longer be worth $\frac{1}{4}$, but $\frac{1}{6}$, & the second clue would be useless; but if one found, that the two circumstances are gathered in the single person of Jacques, the Probability would be changed by persuasion.

651. V. We have examined until here the Probabilities, which have been increased by the contribution of the other Probabilities; there remains to us to speak of the diminution of the Probabilities. This diminution takes place, when the same foundation of the Probability is only probable; because the Probability diminishes, in ratio of the diminution of the Probability of the foundation.

652. If *the Probability of the event* is $\frac{1}{2}$, but if there is $\frac{1}{3}$ of Probability, that the first Probability is able to take place, the Probability of the event will be worth the third of the half, that is to say $\frac{1}{6}$ of Certitude, & *will be*, as one sees, *the product of the multiplication of the given Probabilities.*

653. This rule holds, all the time that a probable event depends on another probable event, whether there are two of them, or a greater number.

654. Paul is joined with nine companions, in the plan of embarking; but of these ten, there has been of them only seven who were entered into the vessel.

When the vessel has quit port, there were discovered two hundred Men, of whom, in the following, one hundred fifty, with two hundred fifty others, had been sent on an expedition, in which these four hundred are all perished, with the exception of thirty. One demands what is the Probability, that Paul is in the number of the dead?

1. The Probability, that Paul has been in the vessel, is worth $\frac{7}{10}$. 2. If he has been there, the Probability that he has been of the expedition is worth $\frac{3}{4}$. 3. If he has been of them, the Probability that he has been killed is worth $\frac{37}{40}$ (603.). The product of these three Probabilities gives the probability that one seeks (652, 653), that is to say $\frac{777}{1600}$, or very nearly $\frac{17}{35}$.

655. The question touching the assurance of the vessel, which must touch at different ports, is precisely of the same nature.

656. A vessel must be rendered to a port, from this port to another; & finally, to return to the place whence it has departed.

The Probabilities of these different navigations being given (615.), one demands the Probability of the return of the vessel; & it appears, by that which we have just said, that it is necessary for this effect only to multiply the Probabilities of the three navigations, considered separately (652, 653).

657. It is by the means of the same method, that one is able to determine the concurrence of the events, which happen at the same time.

658. Four questions are able to take place, in the example of No. 633 & 637. according as the different concurrences of the circumstances.

1. One demands the Probability that there is, that will Pierre & Paul be alive, in ten years?

2. That will Pierre be alone alive;

3. Or that will Paul be alone;

4. Or finally, that both will be dead, in the same space of time?

Among twenty Men of the age of Pierre, three of them die in ten years, & seventeen of them remain alive; therefore the Probability, that Pierre will be alive in ten years, is $\frac{17}{20}$, & that he will be dead, is $\frac{3}{20}$ (603.)

One discovers likewise, that the Probability that Paul will be alive, after the same time, is $\frac{7}{9}$; this which leaves a Probability of $\frac{2}{9}$ for his death (637, 603).

In order to respond to the first question, it is necessary to join, by multiplication, the Probabilities touching the life of each of them in particular, $\frac{17}{20}$ by $\frac{7}{9}$; & the Probability, which results from it, will be $\frac{129}{180}$, that is to say, very nearly $\frac{2}{3}$.

The response to the second question is found, by multiplying the Probability that Pierre will be alive, by the Probability that Paul will be dead, $\frac{17}{20}$ by $\frac{2}{9}$, this which gives $\frac{17}{90}$, that is to say, very nearly $\frac{3}{16}$ (652).

Likewise, $\frac{7}{9}$ by $\frac{3}{20}$ gives a number which satisfies the third question, namely $\frac{7}{60}$, or very nearly $\frac{2}{17}$ (652).

Finally, in order to resolve the fourth question, it is necessary to multiply together the Probabilities, that they both will come to death in the marked time, that is to say $\frac{3}{20}$ by $\frac{2}{9}$; this which gives $\frac{1}{30}$ (652).

This same rule (652.) is able to be applied to every proposition composed of other diverse propositions, which have each their degree of Probability. 659.

One is able to be served by it also in order to determine the Probability of a proof, which is deduced probably from another probable argument; as if Pierre affirmed that he has heard tell a certain fact to Paul, & that the testimony of both was suspect. 660.

It is clear, that a similar concurrence diminishes extremely the Probabilities: because two Probabilities, of which each is worth $\frac{7}{10}$, joined together, would not form completely that which we have called the doubt (652, 596). 661.

Chapter XIX. *Concerning Objections & opposed Probabilities*

It does not suffice, when the concern is to acquire some knowledge, to pay attention only to that which serves to prove what we examine; but it is necessary to consider also, that which one is able to allege in favor of contrary sentiment. 662.

With respect to the mathematical Evidence, it is very true, that one would know nothing opposing; & when there is question of a simple perception, & immediate, he would never know to have error (458). 663.

In some more composite cases , where there is found many judgments, he would no longer have error, when with a single glance we see all these judgments, & when we perceive the liaison that there is between a simple principle, & the proposition of which there is concern (460). 664.

Error takes place when one neglects Evidence (471.); this which, because 665.

of the imperfection of human Understanding, is able to happen in some more composite cases (472, 474): in these cases, one regards as evident that which is not at all such; & one gives place to some objections.

666. It follows thence, that if a proposition appears to me demonstrated evidently, & if one proposes to me, in favor of the contrary proposition, some arguments, which appear to me also evident, it would be necessary for me to remain in doubt; there would be an error, without which I am unable to determine on which side it is found.

667. When from a proposition we are able to deduce evidently a consequence, which is manifestly absurd, we have an undoubtable proof of the falsity of the proposition. But, if this same proposition has a basis, which would appear to us also evidently true, we ourselves find in the case that we just examined, it is necessary to remain in doubt; this is why those who dissent, *this is clearly demonstrated; thus, I am not troubled by the consequences*, reason with as little justice, as those who respond to them, *this proposition leads to absurdity; thus, I am not troubled by the arguments, by which one claims to demonstrate it.*

668. It is an established usage, when there is a question of opposed arguments, & which one would not know how to resolve, that each holds for true the sentiment, of which the demonstration has been proposed to him first; but who does not see, that a similar process repels manifestly from right reason?

669. A commonly received maxim wishes, that one is *dispensed to have regard to the objections, when a proposition is clearly demonstrated.*

670. But this maxim has need of explication. It is false, if by objection we undeerstand an evident argument, in favor of the contrary proposition (666.); in this sense, the contrary of the maxim is true; namely, *that it would not be necessary to have full persuasion, at least all the objections are not raised.*

671. But, it is necessary to distinguish well the objections from the difficulties, which are able to subsist without destroying the same proposition, & which prove, all the more, that is has an obscure side. It is from these sorts of difficulties, of which one is not obliged to be troubled, & to which one must apply the maxim of No. 669.

672. Such is the objection against the movement of the earth about the sun, which has for basis the annual parallax, which has not yet been rather well proved by astronomical observations. But the force of the argument depends on the distance from the fixed stars, which is unknown. By supposing this distance rather great, all the difficulty falls; & we are able to agree that which one says on the annual parallax, without which one has by right to conclude nothing from it against the movement of the earth.

673. One makes many objections against that which the Mathematicians demonstrate on infinity; but one does not prove thence another thing, except that we do not conceive clearly all that which has relation to infinity; this which shakes, in no fashion, the demonstrations of which there is concern.

674. Because each objection, which has for basis our ignorance, has no force that which is evidently proved.

675. Although all these remarks having a direct relation with the mathematical Evidence, one is able however to apply them also to moral Evidence, to the

subject of which we observe beyond this, that *Certitude is never able to be shaken by an opposed Probability* (671.); because this never excludes the contrary Certitude.

But, when Certitude is lacking, although the Probability is great, this last is diminished by an opposed Probability; because a thing, although supported on a great Probability, is able to be false, & that another thing is able to be true, although supported on a lesser Probability (626). 676.

Two Men report to me, on a particular fact, some things directly opposed; if the two testimonies are worthy of faith, I must remain in doubt (666.); & I will conclude simply, that one of the two testimonies is true. 677.

If one of the two is worthy of faith, & the other suspect, I have no regard to the testimony of the last (675.). 678.

If they both are equally suspect, the testimony of the one destroys the one of the other; but if nevertheless the one & the other of these testimonies is probable, because the witnesses are not very suspect, it will be probable that one or the other will have said the truth. 679.

If there is some probable arguments on the two sides, but of which the Probabilities are unequal, one will be able, these Probabilities being given, to determine by how much the one surpasses the other. 680.

But it is not necessary to regard an opposed Probability, that which one deduces from a thing, which has no relation with the subject of which there is concern; *the arguments must be drawn into some convenient sources.* 681.

In order to determine the Probability of a contingent event, we must not pay attention to that which has already arrived in parallel cases; because two events, which are following one another, & of which one depends not at all on the other, have no relation together, than any two other events whatsoever. 682.

The Probability of a roll of dice is not changed by the knowledge that I have of the preceding roll; however, how many people are there not, who pay attention, & who say, *that comes to happen, of which that has not happened a second time at present?* 683.

Of others to the contrary, because a thing, although very probable, has not reused them, no longer dare to be exposed to the same risk, & say, *that is lacking, therefore that will lack again.* 684.

That of chimeras Men are not forged, on the subject of fortune, of happiness, & of misfortune, in the cases where prudence would wish that they paid attention only to the single Probability! 685.

The words of happiness or of misfortune express no another thing, than the relation that there is between the desires of Men, & of the events, of which the Probability is very small. 686.

I will finish that which I had to say on mathematical Evidence, of moral Evidence, & on Probability, by some general observations. 687.

The narrow limits, in which the Intelligence of Men is contained, are the cause that it is not possible for them to avoid all error. They distinguish indeed the rules that it would be necessary to follow for that; but the constant observation of these rules is above human forces. 688.

The opinions of Men depend on the degree of their knowledge, which is found 689.

so different in each of them in particular, & there is no place to be astonished if there is among them so great a diversity of opinions, & if it happens so often to the same Man to change sentiment.

690. One thing appears to me certain: one proposes to me an objection, that I would not know how to resolve; & I begin to doubt (670.) I find the solution of the proposed difficulty, my doubt vanishes; & a second changeover arrives in my persuasion, touching the truth of the thing of which there is concern. Unmatched changeovers arrive less frequently, in some things of great importance, to those which are applied in earnest to extend their knowledge.

691. It is evident, that of all our judgments, there is of them not at all which must suffer more frequent changeovers, than those which revolve on Probability. Each Probability is related to the imperfect knowledge that we have of a subject, & this knowledge is able to vary at each instant (590.). Here is why, the same argument, which to one appears to have nearly no Probability, leaves no doubt in the mind of another, who has some more extended knowledge, on the subject in question (627.).

692. This is what happens, especially, in the things which regard the ordinary course of life. Probability is different for nearly each Man, & depends, most of the time, on the attention that one pays to small circumstances, of which each in particular proves nearly nothing, but which draw their force from their assembly (645.); & which thence form sometimes one so great a Probability, that one no longer distinguishes it from Certitude (627.).

693. We judge from that which is past in the mind of someone, by the times & the place in which he has done certain things; by the attitude & the movement of his body, by his voice, his visage, & the changeovers that we have observed; as also by an infinity of other circumstances, of which the greater part are such, that one would neither know how to make sense of them to another, nor to relate them to some determined measure; this which is a cause that, in these sorts of circumstances, we are able alone to judge of the Probability that they form by their reunion.

694. In order to judge the force of a proof, which results from the union of certain circumstances which concur to the same end, it is necessary from art; & although this art is scarcely able to be reduced to rules, we will not depart from it to say something, in the following.