Studies

in the History of Statistics and Probability

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Notation

Notation **S, G,** *n* refers to downloadable file *n* placed on my website www.sheynin.de which is being diligently copied by Google (Google, Oscar Sheynin, Home. I apply this notation in case of sources either rare or translated by me into English.

General comments on some items

[i, ii] Darboux, who provided an editorial preface to both memoirs, probably glanced over them, saw nothing bad and positively characterized them. I am however critical.

Numerous and quite unnecessary repetitions (most of which I omitted in the translation) testify that Fourier addressed himself to beginners, but he also many times mentioned the regular use of instruments by his readers. And too much is explained in passing, and not on a high enough level. He himself was ignorant of geodetic field work (he obviously never read Gauss) and his relevant descriptions are unsatisfactory, see my Notes including those which preceded memoir [ii].

Fourier highly thought about his study of favourable conditions for indirect measurement. Such conditions had been eagerly sought by ancient astronomers (Sheynin 2017, § 1.14) and their study was resumed by Cotes in 1722. The attempts of such kind constituted the aim of the so-called determinate branch of the theory of errors, nowadays actually taken over by the design of experiments.

The very important novelty [i, § 7] is Fourier's (true, not quite direct) introduction of the notion of true value (see also Sheynin 2007). I have emphasized the relevant phrase.

For additional information I would have recommended the study of Laplace (1814) and Gauss (1809; 1823).

[iv] Aleksandr Grigorievich Obodovsky (1796 – 1852) was a pedagogue and scientist, professor of statistics. The Petersburg Academy of Sciences awarded him the Demidov prize for his book (*faute de mieu*?).

Nowadays we say that the book is devoted to the theory of statecraft (of university statistics) rather than statistics. Achenwall (1749, p. 1) was the first to say that *the so-called statistics is the Staatswissenschaft of separate states* and this opinion persisted; Roslavsky (1841, p. 13) agreed. In addition, Obodovsky (§ 61) called Graunt a political arithmetician which means that he equated it with statistics (actually, with Staatswissenschaft). At least in Germany that discipline, the statecraft, was never forgotten. Today, unlike the olden times, it happily applies numerical data and quantitative considerations, but I am unaware whether it studies medical or criminal statistics or still turns its attention to the boundaries of statecraft and history.

Bibliographic information in the book is utterly bad. Obodovsky names dozens of authors (which proves his erudition), mostly only in Russian, but without the appropriate titles. I have established many *likely* sources (sometimes without dates of publication) and included them in the Author's Bibliography (See *Bibliography*) but did not dare to link them directly with the text.

Contrary to Süssmilch (1758) and ignoring Daniel Bernoulli's 1766 study of smallpox epidemics, Obodovsky (as almost all the other authors of statistical work of later decades) paid no attention to describing the health of population (cf. Note 5 to § 40) although even Leibniz is known to have been interested in public hygiene. Another important subject missed by Obodovsky (just as by later authors) was criminal statistics although he (§ 40) noted that criminality indicated the moral quality of the population.

Then, Obodovsky thought that the study of causes and effects was not really needed (cf. § 54) and he had insufficiently emphasized the value of comparing states or different moments in the life of a given state, although, once more, even Leibniz recommended it (Note 4 to § 25). Finally, there is too many abstract reasoning without justification of the inferences. Cf. Druzhinin (1963) who reprinted a large portion of Obodovsky's book. He maintained, on p. 8, that he, Obodovsky, *scholastically* reasoned about the definition of statistics.

Finally, Obodovsky properly stresses the importance of the theory of statistics, but, just as apparently all statisticians before, say, 1930, he understands it as the means for properly arranging statistics. I follow Pearson (1892, p. 15):

The unity of all [of any given] *science consists alone in its method*.

Then, I maintain that statistical theory or mathematical statistics can be likened to a statistical method with a single specification: theoretical statistics rather than mathematical since only it studies the collection and preliminary investigation of data.

Obodovsky's book is valuable since it provides a picture of statecraft at that time. It also illustrates the well-known fact: except for about the last decade of the bloodiest dictator, Russian scientists knew Western literature; nothing similar can be said about their Western counterparts.

I

J. B. J. Fourier

Memoir on the mean results derived from a large number of observations

Mémoire sur les résultats moyens
déduits d'un grand nombre d'observations (1826).
Oeuvres, t. 2. Paris, 1890, pp. 525 – 545

Note by Editor Gaston Darboux

We see in Arago (1854, p. 360) that during the Second Restauration [1815 – 1830] the prefect of [the department] of Seine, de Chabrol, learned that Fourier, his former professor at the Ecole Polytechnique, was jobless and lived almost without any resources. He conferred Fourier the overall direction of the Bureau de la statistique de la Seine. Arago stated:

Fourier worthily responded. The memoirs by which he enriched the interesting volumes published by the Préfecture de la Seine nowadays serve as a guide for all those who have common sense to see in statistics something else than a mass of undigested numbers and contributions.

The volumes to which Arago referred were the *Recherches* (1821, 1823, 1826, 1829). Each contained memoirs and statistical tables. The unsigned memoirs have been certainly written by Fourier which is testified by his contemporaries. Furthermore, it is sufficient to read *Notions* (1821) and the *Mémoire sur la population* (1823) for detecting the hand and even the style of Fourier.

Led by a sentiment understood by the whole world the illustrious geometer attempted to make way for Chabrol as indicated by Fourier in a passage from the *Hist. de la Acad*. for 1822 (*Mém. Acad. Sci.*, t. 5, p. 314):

The Academy remembers the nice work of Count Chabrol in which he combined the numerous and authentic material and published, in 1821 [the first volume of the] *Recherches …which contained 62 tables. The Academy was interested to find out that the magistrate continues these valuable researches, the only ones up to now of their kind, and that the next volumes* [of that book] *will be appearing incessantly*.

Thanks are due to the administrators who applied their influence and authority of their important activities, and rendered the possible help of every kind for solving the problems which are equally interesting for the government and individuals to the benefit of the *exact sciences and speculations about political economy To proclaim the titles of such contributions which they made known means the discharge of a public debt towards them in the most suitable manner*.

We are content to reproduce here two contributions of quite a general interest [of 1826 and 1829]. There, as clearly as possible, he described the results acquired by science in studying one of the most interesting problems of the calculus of probabilities which should interest to the highest degree physicists and those who are occupied with statistics. In addition, some passages, as that about the Cheops pyramid, indicate that at the time when Fourier participated in the Egyptian campaign he had already known the results which he described and the rules which he announced. The reader can consult Bertrand (1888) about these memoirs[1].

We also mention two other contributions, Fourier (1819; 1822) of the same kind. The Commission which reported in 1822 consisted of S.F. Lacroix, S.-D. Poisson and Fourier, reporter. G. D.

1. The study of the characteristics of the climate, of population and agricultural and commercial riches most often requires the determination of the numerical mean values of certain quantities. We see in the extracts of public registers a large number of different values of such quantities. We collect all the numbers of an experiment […] and obtain the mean value. For example, when undertaking to determine the duration of human life at a certain epoch in a given country we note the age at death for a very large number of men in most various conditions [of life]. The sum of their ages divided by the number of those died is that mean duration of life.

Everyone knows the simple operation of determining a mean number, and, so to say, there is no statistical problem which does not involve the application of the pertinent rule. It is therefore very useful to examine attentively its consequences and the degree of approximation to which it leads.

First of all it is evident that the mean value is known the more precisely [ensures a better approximation] the more is the number of the available observations. It is also seen that it is necessary [in the same problem] to avoid any restrictions to some professions or conditions [of life] but to admit everyone indifferently so that the accidental variations compensate each other in a multitude and variety of the elements. We thus form a mean result in general[2]. In another memoir we will indicate how this compensation is established. It is based on the following principle, which is one of the first theorems in the analysis of probabilities:

Everything accidental and random disappears in an immense number of observations and the multiplicity of chances, and only the certain effect of constant causes is left.

There is no randomness in the natural facts considered in a very large number. I have no intention at all to prove this principle which presents itself to our mind, but I propose to indicate its mathematical consequences and derive usual rules for easily applying them in statistical research.

2. First of all, we recognize that for the same kind of observations the mean result is known the more exactly [ensures a better approximation; this statement repeats a similar phrase in § 1] the larger is the number of the values from which it is calculated. When calculating the mean life, four thousand such values determine it more precisely than only two or three thousand.

But what is the measure of these different degrees of precision, and what relation there exists between it and those numbers? Before solving this problem we ought to remark that we can acquire a rather exact notion of that precision without applying mathematical theories. Suffice it, for example, to separate the totality of very large number of the observed values in two parts, and calculate the mean of each part. If they almost coincide, we are justified to regard them as very precise. Nothing is more proper than this kind of proofs of the precision of statistical results, and it is almost useless to present to the readers the consequences which are not verified by the comparison of mean values[3].

For fruitfully applying this first remark we ought to elevate ourselves to the pertinent principle and imagine quite distinctly that the repetition and the variety of the observations suffice for discovering constant ratios of the effects of unknown causes. This conclusion of which we cite a very simple numerical example is applicable to most diverse objects. In our theme there is no more general and more important notion at all.

3. Suppose that an urn contains unknown numbers [M and N], $M \neq N$, of white and black balls. We can determine their unknown ratio by experience. To this end, we repeat a very large number of extractions of a ball with replacement. We count how many balls of each colour had been extracted, and the ratio of these numbers, m and n, can at first essentially differ from the ratio of M and N, but the former variable ratio m/n will continually approach M/N. The difference between them can be either positive or negative, which is accidental, but it will necessarily have an extremely small absolute value.

Suppose now that, after carrying out this very large number r of trials we repeat an operation of the same kind and make a very large number r_1 of trials. This time the ratio m_1/n_1 will appear and it will also extremely little differ both from m/n and M/N. These differences indefinitely diminish with the increase of r and r_1. These last mentioned numbers can be so large that there will be no appreciable difference between m/n and m_1/n_1.

The verity of these consequences is presented all by itself since common sense suggests it, but mathematical analysis of our time completely confirms it[4]. This analysis determines how many trials ought to be made for becoming practically assured that the second similar operation will provide a sensibly equal result.

This analysis exactly measures the probability of that conclusion. It numerically expresses how probable is the existence of the mean calculated value within given boundaries and it also proves that there exists a boundary of the largest possible errors[5]. These considerations extend to all kinds of research and it is seen that the perseverance and multiplicity of observations as though compensate the ignorance of the causes and is sufficient for discovering the laws obeyed by natural effects. The philosophical sciences are indebted for their progress to Jakob Bernoulli and later great geometers[6].

4. We may apply these principles to the research of the duration of human generations, which is interesting for the natural history of mankind and for chronology, which is not yet at all reduced (réduite) [subjected] to calculus.

First of all we should remark that that duration is not at all the mean life. These two intervals, which many political writers do not at all distinguish, depend on very different conditions. They do not at all consist of the same elements, do not submit in the same way to the influence of civil laws. For example, the law that regulates the marriageable age directly influences the determination of the duration of generations[7]. We also see that it is necessary to consider separately that duration for each sex, for the firstborn, for the succession to the throne.

The common duration of the virile generation is the mean value of the time interval from the birth of a father until the birth of one of his sons. To determine it we ought to obtain a large number of particular values, say three or four thousand, which express that interval. And in each particular case we will know the age which the father had attained when his son was born and calculate the mean value sought.

Obviously, we shall not restrict these calculations to the firstborn since then the result will only express the mean duration of generation of those firstborn which is shorter than the general duration sought. On the contrary, we should admit indistinctly, without any selection, the first, the second, the third etc. sons and pay no attention to special conditions [of life] or profession [cf. Note 2] so as to represent sensibly the general condition of the society by the variety and multiplicity of the observations.

And so, the mean result expresses an approach to the interval sought. However, we still have to determine the degree of approximation. We do not yet know whether the calculated value is very near to the sought value or by how much it can differ from that value. The determination of the pertinent boundaries is important in every research. Until they are known we can only form a very vague idea about the precision of the result. In the following sections we provide an easy rule for measuring that precision.

5. One of the simplest methods of verifying the numbers provided by multiplied observations consists, as I stated above, in randomly separating the series of observations in various parts and comparing the values derived separately in each part. The application of these rules evidently supposes that during the whole interval of the trials the composition of the urn had not changed at all.

We may undoubtedly apply these rules in the case in which the changes were occasioned by the nature of the causes, and thus we can even find out the effect of such changes. But in this case it is necessary to consider separately the intervals during which the cause remains constant and multiply the observations in each of them. The most general source of error and of the uncertainty of its consequences which many authors derive from their research are: **1)** the defects and incomparableness of the initial observations collected by very diverse methods; **2)** too few observations which do not at all permit their separation in series or the separate calculation of the results of each series; **3)** the change, either progressive or irregular, which the causes experience during observation.

6. Until now, we only considered the mathematical consequences and only those that appear at the first study. Now, we ought to investigate this problem more thoroughly and show how they can be explained by analytical theories. If the number of the observed values is very large, and if, upon collecting them we divide their sum by their number, the quotient will be the mean value with a very good approximation.

It is evident that the degree of approximation is the higher the larger is the number of the particular values [similar statements are in §§ 1 and 2]. It is also seen that, if these values very little differ from each other, we are justified in believing that the result is more exact then if these values are very different. And so, the degree of approximation depends not only on the number of the combined magnitudes, it also more or less depends on their diversity. The problem consists in forming an exact idea about that degree of approximation and in showing that the precision of the result is a measurable quantity which we can always express by numbers.

At first we state the rule for deriving that numerical measure of precision. Denote those m particular values by $a, b, c, d, \ldots, n,$ from which we derive the mean result A and suppose that m is very large; calculate g. It serves as a measure of the degree of approximation. The less is g, the nearer is the calculated mean A to the exact sought number. Here are the necessary formulas:

$$A = \frac{1}{m}(a + b + ... + n),\ B = \frac{1}{m}(a^2 + b^2 + ... + n^2),$$

$$g = \sqrt{\frac{2}{m}(B - A^2)}.$$

Suppose, for example, that we have determined 4000 particular values, 1000 equal to 2, 2000, to 5 and 1000, to 12. If all these values were different they could not be grouped, but we only indicate the course of calculations[8]. [Fourier calculates *A, B* and *g* for his example and gets ca. 0.082 as the indicator of the degree of the approximation of the mean.]

7. When describing the real sense of that proposition it is necessary to recall the principle which serves as the basis for the calculations of mean quantities. Suppose that we have collected a large number *m* of observed values and divided their sum by *m* which provides the quantity *A* as the mean value. We had already remarked that, when excluding particular and abstract cases which we will not at all consider, the value of *A* for a very large number of observations will be almost the same for a very large number of other observations.

The mean value derived from an immense number of observations does not therefore change at all. It takes a definite value *H* and we may say that the mean value of an infinite number of observations is a fixed quantity in which nothing random is entering and which has a certain connection with the nature of the observed facts.

It is this quantity *H* which we have in mind as the veritable object of study. When comparing each of the particular values with this quantity we call the differences *errors* or *deviations*. The number *m* cannot be infinite, but it is very large and the mean value *A* is not at all a fixed magnitude *H* but the difference $D = H - A$ is the error of the mean value *A*. It is extremely probable that *D* is very small if *m* is very large. It is capable of taking an infinitely large number of different and very unequally possible values. We will define the probability that the absolute value of *D* does not exceed a given boundary *E*.

8. The probability of an event is known to be estimated by comparing the number of chances favourable for the event with the total number of equally possible chances[9]. Put *M* balls of different kind in an urn, *m* of the first kind, *n*, of the second kind etc. Then *m/M, n/M*, … will be the probabilities of extracting balls of the respective kind.

This is the only way in which the solution of all problems in the analysis of probabilities is presented. However complicated is an exactly defined event, its probability can be measured since it is possible to prove that it is the same as an extraction of a ball of a certain kind from an urn which only contains *m* balls of that kind out of *M* balls of various kinds[10].

The fraction *m/M* is the measure of the probability sought. The entire art of the research consists in deriving the conditions concerning the value *m/M*. However, it often happens that this mathematical deduction is difficult and requires a deep knowledge of the science of calculus.

9. That notion of probability is applicable to errors of measurement to which the use of instruments is subjected. However precise is a certain instrument, say for measuring angles, at first we only get an approximate value of an angle. Its error is likely very small, but the contrary is not impossible. It is only very probable that the absolute error does not exceed a certain boundary, for example 3 arc minutes[11]. This is the case even with very imperfect instruments, and a small error of one minute is much more probable.

The instrument can be such that the mean value of its error at each operation is one minute. For a better instrument the mean (*moyenne*) error is less than one minute. It is possible to have an instrument of that kind whose mean error is five times less, and then we say that it is five times more precise than the previous[12].

10. It is easy to extend the research of mean values onto a mathematical definition of the degree of approximation. The mean value *A* derived from *m* observations can differ from *H* and the difference is likely very small if *m* is very large. The eventual error is susceptible to infinitely many unequally possible values. It is extremely probable that the absolute error does not exceed a certain quantity. There exist other boundaries less remote from each other for which the probability of error is only 1/2.

In general, the determination of the mean result of a large number of particular values involves the measurement of a quantity with an instrument whose precision we may increase as much as needed[13] and increase indefinitely the number of observations. It is easy to compare, according to the rule in § 6, the precision of *A* derived by a large number *m* of observations with A_1 derived by m_1 observations. According to that rule [Fourier explains how to calculate *g*]. In the same way we calculate g_1. It is rigorously demonstrated according to the principles of the calculus that the degree of approximation entirely depends on *g*. It is the higher the less is *g*. The precisions of *A* and A_1 are in the inverse ratio of *g* and g_1. We ought to remark that this comparison does not even suppose that the observations were of the same nature since it is purely numerical and therefore most diverse researches can be viewed from that common point of view.

11. To end the discussion we should determine the probability that H, the quantity sought, is contained within the proposed boundaries, $A + D$ and $A - D$. The following table shows the probability P of an absolute error larger than D and expressed as a product of g by proposed factors δ.

0.47708	1/2	1.38591	1/20	1.98495	1/200
2.46130	1/2000	2.86783	1/20,000		

The values of P[14] show the probability that the exact value of H is contained within the boundaries $A + g\delta$ and $A - g\delta$. […] Finally, it is possible to bet much more than 20,000 to 1 that the error of the mean result is less than $3g$.

In § 6 we derived a mean result 6 and it can be considered certain that this value does not err by more than ca. 0.082 which is the rule for the value of g. And so, the quantity sought, H, is contained within 6 – 0.246 and 6 + 0.246.

12. To facilitate the application of the rule for calculating g we remark that it is possible to subtract from each observed value a, b, c, … a common quantity u and operate with $a - u, b - u, c - u$, …We will always get the same g as previously. For example, we may subtract a common quantity 2 from the given data of § 6 and get

$$1000\cdot 0 + 2000\cdot 3 + 1000\cdot 10 = 16{,}000$$

as the sum of the values so that $A^2 = 16$ […] and get the same answer [for one of the intermediate numbers] 13 1/2. This takes place for any u.

Moreover, we may consider that particular values almost equal to one another can be considered equal and the calculation will be much easier.

In these researches of mean results the main aim is to investigate whether they approximate to a proper degree and to form a good idea about that degree. It is less important to calculate an entirely exact value of the probability of errors than to prove that the quantity sought is contained within very near boundaries and to compare the probability of that latter conclusion with the probability which we determine for the most essential facts of life[15].

Therefore, it is not necessary at all, when applying the previous rule, to pay attention to very small differences between two observed values. It is possible without a sensible error to suppose that they are equal. In addition, if the interest in this problem requires it, we direct the calculations so that the consequences are applied all the more to the observed precise values.

Considerations of that kind merit all our attention because they concern most problems of the analysis of probabilities and essentially facilitate applications. We have already made use of them in the theory of assurances[16].

13. We do not wish at all to describe the analytical demonstration of the rule of § 6 since it demands the use of mathematical formulas. It may be regretted that the just as generally applied usual rule does not admit a simpler demonstration, but this fact is occasioned by the essence itself of the problem.

That same rule can be presented in another useful form which indicates its connection with known rules. [Fourier explains the steps of the calculation of g in the form]

$$g = \frac{1}{m}\sqrt{2[(a-A)^2 + (b-A)^2 + (c-A)^2 + ...]}\ .]$$

For example [a numerical example follows, once more with thousands of equal particular values].

14. In the example of § 6 we got $g = 0.08216$, now we have $g_1 = 0.01962$ which is much better [shows a much better result]. Applying the second example to the table of § 11, we see the probability of the absolute error not exceeding certain boundaries which are the products of g_1 by the factors 0.47708 and 1.38591.

It follows that the probability that the error of the mean result, of 6, in the first example, does not exceed $0.47708g$ is equal to the probability that the error of the mean result, 13 1/2, in the second example, does not exceed $0.47708g$ [g_1] and that both probabilities equal 1/2. The probability of an error larger than $1.38591g$ in the first example and larger than $1.38591g$ [g_1] in the second example are also equal and both probabilities equal 1/20.

In general, the probabilities of an error larger than some boundary Δ and Δ_1 in those cases are equal if $\Delta/\Delta_1 = g/g_1$. And so, if we wish to compare the precision of the mean results in two investigations it is sufficient to calculate g and g_1 and compare them. […]

15. The rule of § 6 immediately provides the precision of the mean result multiplied by the square root of the number of observations. For a large number of observations m we may consider the value of the mean result as an invariable quantity quite independent from that number and only dependent on their size. The same applies to the mean of the squares of those values: the difference between the mean value and A^2 is sensibly independent from the number of observations. By dividing the square root of the double of that difference by $\sqrt{m}$ we get the normal value g (?) and $3g$ is the boundary of the largest errors. We see that the largest possible error decreases with the increase of m in the inverse ratio of $\sqrt{m}$.

As to the error with probability 1/2, we know that it is invariably proportional to g just as any other error whose probability is known [is fixed]. Therefore, for the same research the precision of the mean result changes as the number of the observed values increases. It doubles when that number becomes four times larger, it trebles when that number increases nine times etc.

This consequence is simple and remarkable and it ought to be known to all those who carries out statistical researches. […]

16. This memoir describes the application of the known theories to one of the fundamental problems of statistics. For indicating the totality of the propositions contained here, I provide below a summary of each article [section].

As to the general conclusion, it can be formulated thus:

After deriving the mean result A of a large number m of partial values, it is still required to evaluate the degree of approximation [to the true value of the unknown] and it is therefore necessary to [calculate g] which is a measure of the degree of approximation. The precision of the result is the inverse ratio of g. The error of the result is positive or negative but, anyway, in practice we should regard as certain that that error is less than $3g$. […]

Summary of the articles [sections]

1. The aim of the memoir is to provide a usual and general rule for estimating the precision of the mean result.

2. The degree of approximation can be indicated by comparing two mean values derived from different series of observation.

3. Experience based on numerous and very differing observations can easily indicate the laws of phenomena whose cause is unknown.

4. A remark about the calculation of the duration of human generations.

5. Necessary conditions for the precision of the research of this kind.

6. Description of the rule which provides the measure of the degree of approximation.

7. Mathematical definition of the error of the mean result.

8. The general form of all the solutions which are derived by the analysis of probabilities.

9. Errors of measurement with instruments. Definition of the mean error.

10. The same notions are applied to errors of the mean results.

11. We can determine the probability that the error of the mean result is contained within proposed boundaries. A pertinent table.

12. On the facility of applying the rule of § 6: 1) the subtraction of a common quantity from each particular value; 2) unification of values which very little differ from each other. General remark about the application of the calculus of probabilities.

13. It is also possible to find out the measure of the degree of approximation by [calculating *g*] as defined in § 6.

14. The fraction 1/*g* is the exact measure of the precision of the mean result.

15. This precision increases as the square root of the number of the observed values.

16. Summary and conclusions.

II

J. B. J. Fourier

Second memoire on mean values and errors of observation

Second mémoire sur les résultats moyens et sur les erreurs des mesures (1829). *Oeuvres*, t. 2. Paris, 1890, pp. 550 – 590

Notes by translator.

1. Fourier numbered his formulas in the worst possible manner: he repeated the same numbers in several sections; thus, formula (1) occurs in §§ 3, 7 and is mentioned in 9. Other inadmissible cases are seen below, where I decided to collect all the formulas of this memoir. When referring to them, Fourier does not mention the pertinent section and thus worsens the situation still more. I left the author's numeration but added the numbers of the sections; thus, formula (2) became either (2.3) or (2.11) etc. The numbers of formulas (5.7), (1.25) and (1.27) are my own; Fourier had not numbered them.

$$x = F(a, b, c, \dots) \qquad (1.3)$$

$$dx = F_1(a, b, c, \dots)da + F_2(a, b, c, \dots)db + F_3(a, b, c, \dots)dc + \dots \qquad (2.3)$$

$$\mathrm{D}x = \sqrt{(F_1\mathrm{D}a)^2 + (F_2\mathrm{D}b)^2 + (F_3\mathrm{D}c)^2 + \dots}\,. \qquad (3.4)$$

(3.6) = (3.4)

$$dx = \sqrt{(F_1 da)^2 + (F_2 db)^2 + (F_3 dc)^2 + \dots} \qquad (4.6)$$

$$x = a + b + c + \dots \qquad (1.7)$$

$$dx = da + db + dc + \dots \qquad (2.7)$$

$$\mathrm{D}x = \sqrt{\mathrm{D}a^2 + \mathrm{D}b^2 + \mathrm{D}c^2 + \dots} \qquad (3.7)$$

$$dx = \sqrt{da^2 + db^2 + dc^2 + \dots} \qquad (4.7)$$

$$a - \mathrm{D}a,\ a + \mathrm{D}a;\ b - \mathrm{D}b,\ b + \mathrm{D}b;\ c - \mathrm{D}c,\ c + \mathrm{D}c;\ \dots \qquad (5.7)$$

(1.9) = (1.3)

$$\mathrm{D}x = F_1\mathrm{D}a + F_2\mathrm{D}b + F_3\mathrm{D}c + \dots \qquad (2.11)$$

$\mathrm{D}x = 2a\mathrm{D}a + 2b\mathrm{D}b + 2c\mathrm{D}c + \dots$ (2a.11)

The number (2.11) was mistakenly repeated, so I added a letter.

$$Dx = \pm\sqrt{(2aDa)^2 + (2bDb)^2 + (2cDc)^2 + \dots} \quad (3.11)$$

(3.12) = (3.4)

(2.17) = (2.3)

$$Dx = \sqrt{b^2c^2Da^2 + a^2c^2Db^2 + a^2b^2Dc^2}. \quad (3.19)$$

$$\frac{dx}{x} = \frac{da}{a} + \frac{db}{b} + \frac{dc}{c}. \quad (e.20)$$

$x = b\tan\alpha$, $\ln x = \ln b + \ln\tan\alpha$, (1.22)

$$\frac{dx}{x} = \frac{db}{b} + \frac{d\alpha}{\sin\alpha\cos\alpha}. \quad (2.22)$$

$$\frac{Dx}{x} = \sqrt{(\frac{Db}{b})^2 + (\frac{D\alpha}{\sin\alpha\cos\alpha})^2} = \sqrt{(\frac{Db}{b})^2 + (\frac{2D\alpha}{\sin 2\alpha})^2}. \quad (1.25)$$

$$D_x = \sqrt{(D_aF)^2 + (D_bF)^2 + (D_cF)^2 + \dots}. \quad (1.27)$$

2. Fourier repeatedly formulated some statements. I write them down here and do not include them in the translation. And I also explain here some circumstances.

Mean error: defined in his memoir [i, § 9]. Actually, however, it is the probable error, see Note 12. He referred to that memoir by letter M, which added difficulties to his readers. I replaced M by [i].

Absolute value of a number: Fourier applies this term only once or twice; instead, he writes in *either direction* or *negative or positive.* I invariably replace these expressions by *absolute value*.

Statements: If the probability of event *a* is $P = 1/2$, then, in the long run, the frequencies of *a* occurring and failing are approximately equal and their ratio is tending to unity. Low probabilities ought to be neglected; his choice: *low* means $P < 1/20.000$. However, the appearance of events having such low probabilities is not excluded.

The boundaries within which event *a* is practically always situated are $a - 3g$, $a + 3g$, $g = \sigma/\sqrt{2}$ and the generally known rule of *three* *sigma* (whose shortcomings are also known, see Helmert 1877) is here replaced by $3g = 2.1\sigma$ with the same shortcomings.

Relative error: I omit the explanation given by Fourier.

Notation: *da, db, ... dx* and *Da, Db, ... Dx* (sometimes D_a, D_b, …, D_x; these changes likely occur randomly, owing to negligence). General meaning: errors, but D*a*, … sometimes mean maximal errors, which Fourier does not invariably indicate.

1. Statement of problem: to discover on which law of error depend the partial errors of the observations. The third volume [1826 of the *Recherches statistiques* …] contains a rule for estimating the precision of the mean results derived from a large number of observations. I propose to complete here its application and to describe a method of the same kind for the results of the calculation for some number of magnitudes of a different nature.

In the application of mathematical sciences the unknown magnitudes which we need to determine are not entirely fixed; they are only very nearly approaching those fixed values. The errors which are impossible to avoid are contained in certain boundaries and it is very important to know these. It may be said that any application of the calculus is vague and uncertain if we are unable to estimate the extent of the error by which the result can be affected.

[…] An unknown *x* can be a certain function of three different data, *a, b* and *c*. We operate on them in a way whose essence is supposed to be known since it depends on the type of that function. For example, [Fourier describes an intersection, a French (and English) term which he apparently did not know]. The essence of the function can be much simpler. Thus, for determining the volume of a rectangular prism whose dimensions are *a, b* and c, we simply form the product *abc*. These examples provide a sound idea about the object of our investigation.

Suppose in general that some number of known magnitudes *a, b, c, d*, … are measured[17] and that it is required to determine an unknown value *x* which is a certain function of those magnitudes. The type of the function is known which means that we know how to calculate *x*. It is required to find out how the inevitable errors can influence the error of *x*.

2. Examples of elucidating the essence of this problem. However carefully the given magnitudes are measured, they are evidently always exposed to errors which are the larger the less precise are the instruments. We have seen in the cited memoir [i][18] that, when multiplying the observations and taking the mean (moyenne) value of the results, we can indefinitely diminish its error. If the number of observations is rather large, we get the mean result which can only be affected by a very small error. And, what is very important, we will know by applying the general rule the mean error of that result. This error has probability 1/2 according to our definition [i, § 11].

Now, before beginning the actual research, it is necessary to recall very clearly that definition and its consequences described in §§ 11, 14 and 15 of that memoir.

And so, suppose that by applying these principles (?) and the derived general rule we found out for each of the given magnitudes its mean error. At the same time we determine the boundaries of the largest possible error[19]. Actually (§ 11) $3g$ exceeds the largest possible positive or negative error, since among more than 20 thousand chances only one leads to that error. And calculations provide the value of g. So $3g$ is the boundary of the largest error. By multiplying g by 0.47708 we determine the mean error.

When determining [by intersection] an unknown magnitude whose value is a certain function of a horizontal base and of two angles we may suppose that each of these angles is measured by a very large number of observations as also is the length of this base. So we know for each of these magnitudes a, b, c the mean error whose probability according to the definition of [i, § 9] is 1/2. For example, let the estimate of the mean error of each angle, a, b, be 1 minute so that it is equally possible that those errors are larger or smaller than that. And suppose that the mean error of the base is 1 *cm*. […] Therefore[20], the error of the unknown x is a function of three partial errors which are made when measuring a, b and c.

The error in the measurement of each of these angles somehow influences the error of the calculated result as also does the error of the base. The problem which we have in mind consists in examining how these errors can influence x. […] The number of the given magnitudes can be much larger than three. For example, if we measure the difference of the levels [of the heights above the mean sea level] of two points distant from each other we divide the total distance into a certain number of parts and measure the difference of the levels [in each part] by the instrument and thus determine the required difference. […] It is mainly to operations of this kind that it is proposed to apply the rule which is the actual object of our research.

In general, suppose that a certain quantity x is a function $F(a, b, c, d, \ldots)$ of many given magnitudes $a, b, c, d, \ldots$ and the type of this function is known. For example, suffice it for determining x to add up all these measured magnitudes but in general we ought to deal in a certain way with all these magnitudes so that the result of our last operation is the value of x.

And so, we suppose that for each of these magnitudes we know by experience the possible mean error or the boundaries within which that error is certainly contained. It is required to determine the mean error of the result of calculations and the boundaries within which the error of the result is certainly contained.

3. The differential expression of the error of the calculated result. It is not at all sufficient for answering the question which should be proposed. Before announcing the general rule which should be followed for determining the mean error of the result of the calculation it is necessary to note the possible influence of each partial error. Mathematical analysis easily solves this latter question by the differential method. However, since this analytical expression is not at all generally known, we provide below a practical rule whose application is very easy and always, in all possible cases, leads to the same result.

Here, first of all, is the analytical expression. Denote by $da, db, dc,$ ... the errors of each of those partial observations. Then $F(a, b, c, \ldots)$ represents the known function which expresses the value of x. It indicates a sequence of operations necessary to perform on the magnitudes $a, b, c, \ldots$ so that the end result will be x.

We differentiate the equation

$$x = F(a, b, c, \ldots) \qquad (1.3)$$

with respect to each of the variables $a, b, c, \ldots$:

$$dx = F_1(a, b, c, \ldots)da + F_2(a, b, c, \ldots)db + F_3(a, b, c, \ldots)dc + \ldots \qquad (2.3)$$

The coefficients $F_1, F_2, F_3, \ldots$ are functions of those same variables $a, b, c, \ldots$ and their numerical values can be calculated since we attach to their arguments the respective values provided by observation. The factors $da, db, dc, \ldots$ represent very small quantities, the errors made when measuring $a, b, c, \ldots$ For example, one of the measured magnitudes a, is the segment chosen as the base, and its error is 1 *cm*. If a is an angle, its error da can be represented as

$$da = \frac{\pi}{180 \cdot 60}$$

or 1′. Equation (2.3) can show the error of the unknown x if the respective errors $da, db, dc, \ldots$ are known.

But we will solve quite another problem since those errors are unknown. By repeatedly using our instrument we only know that they cannot exceed some boundaries and we establish that the exact values of these errors are contained between those boundaries. We find them by adding a very small quantity Da to, and subtracting it from the measured quantity of a.

For the error of x we ought to find a small magnitude Dx similar to Da and become assured that x is contained between $x - Dx$ and $x + Dx$[21].

4. The general rule which solves the latter problem, the calculation of the boundaries of the error. After solving that latter problem by an exact analysis we come to a general rule expressed by the following equation

$$\mathrm{D}x = \sqrt{(F_1\mathrm{D}a)^2 + (F_2\mathrm{D}b)^2 + (F_3\mathrm{D}c)^2 + \ldots}\,. \qquad (3.4)$$

The numerical values of F_1, F_2, F_3, … are known and the small magnitudes Da, Db, Dc, … are known as well since the repeated use of the instrument shows us that the errors of a, b, c, … are contained between their boundaries (5.7).

And then we determine Dx by formula (3.4). The exact value of x will not differ from the value provided by equation (1.3) more than by the absolute value of Dx.

We have remarked that the boundaries Da, Db, Dc, … are indicated by the repeated use of the instrument. Actually, after calculating g (§ 9 and the next ones in memoir [i]) we will assume 3g as Da. We apply similar calculations to b and obtain Db etc. and thus get Dx by formula (3.4).

5. Application of the same rule for calculating the mean error. Consider now the mean value and denote by da the mean error which characterizes the measured quantity a. That error is known to depend on g. Indeed, 0.47708g is that mean error da (§ 10) In the same way we deal with b, c, … and determine the mean errors db, dc, … instead of Da, Db, Dc,… And now we only have to insert da, db, dc, … instead of Da, Db, Dc, … in formula (3.4) and we will know the mean error which may be attributed to the unknown x.

6. Remarks about the use of that rule. An exact description of its consequences. The preceding analysis is reduced to the following propositions. The equation (1.3) provides, as stipulated by the hypothesis, the value of x as a function of the measured and therefore known quantities a, b, c, … corrupted by small errors da, db, dc, … If those errors are known, the differential equation (2.3) provides the error dx after inserting the measured values of a, b, c, … The equation (3.4) represents the value of x [see below] as expressed by equation (1.3). It is corrupted by a certain error Dx and the exact value of x is contained between x – Dx and x + Dx whereas Da, Db, Dc, … denote the largest possible errors in a, b, c, … Finally,

$$dx = \sqrt{(F_1 da)^2 + (F_2 db)^2 + (F_3 dc)^2 + \ldots} \qquad (4.6)$$

represents the mean error of the unknown x if $da, db, dc, \dots$ are the mean errors of those measured quantities.

Equations (3.4) and (4.6) show the very useful and very general consequences of applying the calculus. We should never forget the definition of the mean error and remember that the mean partial error da has probability 1/2.

7. Application to the case in which the unknown is the sum of the measured quantities. To completely describe the consequences of equations (2.3), (3.4) and (4.6) it is convenient to multiply the examples. Those which we choose at least present some applications as well.

The simplest case is that in which the unknown is the sum or the difference of the measured quantities. It occurs when levelling and in general when the unknown is divided into many parts and each is measured separately.

Suppose therefore that the function is

$$F(a, b, c, d, e, f, \dots) = a + b + c + d + e + f + \dots$$

Equation (1.3) becomes

$$x = a + b + c + \dots \qquad (1.7)$$

and, after differentiation,

$$dx = da + db + dc + \dots \qquad (2.7)$$

Now, Da, Db, Dc, … are the respective boundaries of the largest possible errors in $a, b, c, \dots$ and

$$\mathrm{D}x = \sqrt{\mathrm{D}a^2 + \mathrm{D}b^2 + \mathrm{D}c^2 + \dots} \qquad (3.7)$$

If $da, db, dc, \dots$ are the mean errors and $a, b, c, \dots$ are known, then

$$dx = \sqrt{da^2 + db^2 + dc^2 + \dots} \qquad (4.7)$$

We ought to imagine that the repeated use of the instrument indicates that the largest possible errors in $a, b, c, \dots$ are contained within boundaries

$$a - \mathrm{D}a,\ a + \mathrm{D}a;\ b - \mathrm{D}b,\ b + \mathrm{D}b;\ c - \mathrm{D}c,\ c + \mathrm{D}c;\ \dots \qquad (5.7)$$

so that equation (3.7) leads to the boundaries of x:

$$a+b+c+\sqrt{\mathrm{D}a^2+\mathrm{D}b^2+\mathrm{D}c^2+...},\ a+b+c-\sqrt{\mathrm{D}a^2+\mathrm{D}b^2+\mathrm{D}c^2+...}\ .$$

8. A remark about the result which follows if only the boundaries of the partial errors are considered. By hypothesis the largest possible error of *a* is less than the absolute value[22] of D*a* and we conclude that *a* is certainly contained within *a* – D*a* and *a* + D*a* and that a similar statement about *b, c, d*, … will take place. Therefore, *x* is certainly contained within

$$a + b + c + \ldots\ \mathrm{D}a\ \ \mathrm{D}b\ \ \mathrm{D}c + \ldots,\ a + b + c + \ldots + \mathrm{D}a + \mathrm{D}b + \mathrm{D}c + \ldots$$

However, when the measured quantities *a, b, c, d, e, f*, … are very numerous, the interval between the boundaries is exaggerated. This does not concern equation (3.7) in which the boundaries are (5.7). This means that the probability of *x* not exceeding D*x* as provided by equation (3.7) coincides with the probability that *a* does not exceed the absolute value of D*a*.

And the possibility of an absolute error in *x* larger than the right side of (3.7) is exactly the same as the possibility of an error in *a* larger than D*a*. The probabilities of both events coincide and are lower than 1/20,000. […]

We come to the same conclusion when considering the mean errors *da, db, dc*, … Actually, for deriving these small quantities we calculate separately the values of *g* which correspond to these quantities and multiply them by the same factor, 0.47708. This is the rule for calculating the mean error.

But the probability of the mean error *da* is the same as that of *db* or *dc*. That common probability is 1/2 because the analytical principles on which the equation (4.7) was derived prove that the probability of making a positive or negative error

$$\sqrt{da^2+db^2+dc^2+...}$$

by assuming that *x* is the sum in equation (1.7) is the same as for the partial errors *da, db, dc*, …

9. Expression of the mean error in the general case. In general, when applying equation (4.7) for estimating the error which can be made when assuming equation (1.3), we find that (4.6) expresses the mean error *dx* […]

And so, when applying the formula (1.3) very many times[23], we will always make an error when determining *x* because of the errors in the measurement of *a, b, c*, … On the other hand, we may thus determine, by applying the general rule of memoir [i] and derive formula (4.6). That error is either larger or smaller than dx.

10. The measure of the probability of any error. If we multiply g not by 0.47708, but by another common factor we arrive at another expression for the error of the result. This error will not be dx at all and its probability will not be 1/2, it will be P [i, § 11]. […]

11. The error deduced from the differential expression is exaggerated. A particular example indicates the truth of this remark. We say that when attributing to the partial errors Da, Db, Dc, … the largest possible values admitted by the known essence of the instrument which serves for the measurement, we easily find the boundaries of the errors when assuming equality (1.3).

However, these boundaries are too widely apart and much differ from the results described here. Actually, when applying the differential equation

$$\mathrm{D}x = F_1\mathrm{D}a + F_2\mathrm{D}b + F_3\mathrm{D}c + \ldots \qquad (2.11)$$

and attributing to Da, Db, Dc, … their largest possible absolute values it becomes certain that the right side of (2.11) provides the boundaries which exceed the largest possible absolute error.

We are therefore assured that each possible error of x is contained within those boundaries. For example, suppose that

$$F(a, b, c, \ldots) = a^2 + b^2 + c^2 + \ldots$$

Then (2.11) becomes [a formula with the same number! I added the letter a]

$$\mathrm{D}x = 2a\mathrm{D}a + 2b\mathrm{D}b + 2c\mathrm{D}c + \ldots \qquad (2a.11)$$

If we actually know by experience with the instrument that the absolute error never exceeds Da and if we also know the largest possible errors Db, Dc, … of b, c, … we will substitute in equation (2a.11) the extreme values Da, Db, Dc, … which render all the terms positive and then negative.

Now, choosing for Dx the sums of all positive and then negative terms we get the boundaries within which the eventual value of Dx is necessarily contained. Thus we find the boundaries of the errors of determining x, that is, of assuming that

$$x = a^2 + b^2 + c^2 + \ldots$$

Those are the boundaries of the errors determined by equation (2a.11). However, without making any use of the analysis of probabilities we badly estimate the precision of the result. It is rather better to abandon equation (2a.11).

We should draw together those boundaries and conclude that the result is contained within those new boundaries. This is the conclusion of equation

$$Dx = \pm\sqrt{(2aDa)^2 + (2bDb)^2 + (2cDc)^2 + ...} \qquad (3.11)$$

Comparing it with equation (2a.11) we see that instead of taking the sum of the terms assuming that each is positive, we choose the square root of the terms of the squares. This final result is always less in absolute value than the sum of all the positive terms.

12. That last consequence is general. A figure which shows that that conclusion is very sensible. In the general case the differential equation (2a.11) is

$$dx = F_1 da + F_2 db + F_3 dc + \ldots$$

and in that case the equation (3.11) is

$$Dx = \sqrt{(F_1 da)^2 + (F_2 db)^2 + (F_3 dc)^2 + ...}$$

This equation differs from (2.11) in that the sum of the terms $F_1 da$, $F_2 db$, ... which we suppose positive are replaced by the square root of the sum of their squares. A simple construction makes this last consequence sensible[24]. At an extremity of a segment 01 of length a we raise a perpendicular 12 of length b. Then draw a perpendicular to 02, a segment 23 of length c etc. The square of 02 is equal to $a^2 + b^2$; the square of 03 is equal to $a^2 + b^2 + c^2$; of 04, $a^2 + b^2 + c^2 + d^2$, continued indefinitely. The perimeter 01234 is the sum $a + b + c + d$ and the diagonal 04 is equal to the square root of the sum of the squares $\sqrt{a^2 + b^2 + c^2 + d^2}$.

The right side of equation (3.4) always has an absolute value less than the right side of equation (3.11) since the values of Da, Db, Dc, … are the same and one of the quantities is the perimeter 01234, the other, the diagonal 04. Therefore, the application of equation (2.11) does not provide a real knowledge of the precision of the results. It is equation (3.11) which should be applied for finding out the boundaries of the calculated results. Indeed, it rigorously corresponds to the boundaries of the largest errors whose measured quantities are perhaps affected [errors made in the measurement]. Nevertheless, it is not futile to consider the extreme boundaries provided by equation (2.11) since it provides the first approximate knowledge of the possible error of the result. Then we get the exact expression by taking the square root of the sum of squares of the terms.

13. The same analysis applied to estimate the boundary of the error of measuring a length consisting of many parts. The general result. The formulated problem which we consider as an example occurs in practice, and it is quite proper for describing the application of the principles. The analysis which serves to determine the expression of the mean error reduces to the following problem whose consequences are remarkable. When we measure a magnitude which consists of a large number of parts by measuring each part, the error of the total result depends on the partial errors according to some law which we should discover.

We suppose that when measuring each part we can err in either direction by a certain quantity the boundaries of which are known by experience. We also take into account that, having a considerable number of these parts, we should not think that all the partial errors are of the same sign. On the contrary, it is extremely likely that it is equally easy to make negative and positive errors. When the number of partial errors is large, there establishes a compensation which tends to diminish the total error.

It is not rigorously impossible that the partial errors, even in a very large number, are all positive or all negative, but we should not at all suppose that such an event takes place because its probability is very low. This probability is comparable [is equal] to that of many events about which we know that they are not entirely impossible but so feeble that in ordinary usage no reasonable man admits it as a motive for his actions.

Now we should estimate the total error which we justifiably fear when adding up the measurements of the separate parts. We suppose that each partial error can equally be either positive or negative and that we know by repeated experience that that error is contained within certain boundaries. We have to determine the corresponding boundaries of the total error. They should be such that we will believe them as much as we believe in the similar case about the partial errors.

Mathematical analysis completely solves this problem, and here is the result: Denote by e the boundary of the absolute error of a partial measurement. It should be multiplied by the square root of the number (n) of the parts and the product will be exactly that sought boundary. Denote the total result by a. We are equally justified in believing that that result is contained within the boundaries $a + e\sqrt{n}$ and $a - e\sqrt{n}$ as that each partial result is contained within that same result $\pm e$. It is a grave error to consider that the total result is contained between the boundaries $a + ne$ and $a - ne$. These boundaries are too distant from each other and provide a very imperfect knowledge of the precision of the result.

14. Example of the last problem. This problem is often encountered in the applications, for example when we wished to determine the height of the Cheops pyramid[25]. The construction of this exceptional monument allows to measure separately each step [of the staircase], and those charged with that task knew by experience the boundary of the possible error of a partial measure.

It was required to estimate in advance the degree of precision provided by that process and this problem was solved by the analysis of chances[26]. It was sufficient to multiply the boundary of the partial error by 14 since the number of steps was 203 $[\sqrt{203} = 14.2]$. After this calculation the result was compared with the result of a trigonometric measurement [of an intersection] and a singular conformity between the two methods was found. And it was possible to be surprised that that analytical procedure was not done previously. The same consequences apply for a great levelling, for the measurement of geodetic bases and other problems of the same kind.

15. Differential coefficients which measure the influence of each partial error on the error of the result. We will now describe the general rules for estimating the precision of the results of calculation. Above, we provided the differential expressions of the error of our arbitrary function $F(a, b, c, \ldots)$ of many measured quantities $a, b, c, \ldots$, (2.3).

We ought to remark that each coefficient such as F_1 numerically reveals how the error of measurement da influences dx. The larger is that coefficient the more does it influence dx. The consideration of those coefficients is therefore important. It is very useful to know separately the influence of each of the errors since we thus distinguish which of those data should be more necessary to measure very precisely.

16. A practical principle which easily indicates the first part of the error of the result and the pertinent differential coefficient. The application of the differential analysis seems remote from the main aim, from rendering the applications easy and useful. However, we can happily supplement that usage. The function $F(a, b, c, \ldots)$ is known by hypothesis and we know how to deal with the data $a, b, c, \ldots$ for determining x whose value indeed is $F(a, b, c, \ldots)$. And so we attach first of all the immediately observed values to $a, b, c, \ldots$ and calculate F. That is the first result. Then we vary one of those data, for example, a by a very small quantity. If a is a length we add a centimetre and repeat the preceding operation without changing the other quantities $b, c, d, \ldots$ at all. That new operation provides the second result little differing from F and we denote it by $F + D_aF$. That little difference, D_aF, expresses the effect of the change of a alone by a centimetre on the error of x. [F_1 also becomes $F_1(a + \Delta a, b, c, \ldots]$.

For the aim of our research it suffices to find D_aF since it is the first term of the value of dx as provided by the equation (2.11).

17. That same rule reveals all the parts of the error of the result and all the pertaining differential coefficients. In the same way we find the other parts of dx. Thus, for $F_2(a, b, c, \ldots)db$ we vary b by a small quantity D_b (for example, by 1 minute or its fraction if b is an angle or by 1 *cm* or a part of a centimetre if b is a length). And so, we have $b + D_b$ and calculate $F(a, b + D_b, c, \ldots)$ which little differs from $F(a, b, c, \ldots)$.

Denote by D_bF the very small difference between the two values of F and calculate the second part of dx caused by D_b. That part is $F_2(a, b, c, \ldots)D_b$, i. e., the part of dx caused by the error D_b. By dividing the calculated increment by D_b we get F_2, if needed, but it is sufficient to consider FD_b. So we vary only c by a small quantity D_c (which we previously considered arbitrary) and get the third term $F_3(a, b, c, \ldots)D_c$ which is the third part of dx.

And so we find successively all the terms which constitute dx in equation (2.3).

18. The square root of the sum of the squares of the terms calculated by the previous rule allows us to find the boundaries of the largest error of the unknown and its mean error. According to that calculation we should successively vary $a, b, c, \ldots$ by small and as previously regarded, arbitrary quantities. For example, we have the largest possible error D_a in the measurement of a and similarly the boundaries for $b, c, \ldots$ Those boundaries are the same as were found by the application of the general rule provided in memoir [i] for exactly estimating the precision of the mean results. They coincided with those which entered equation (3.11) and have been thus denoted by D_a, D_b, D_c, … in § 4. We can also choose for D_a, D_b, D_c, … the mean errors $da, db, dc, \ldots$ (also determined by the general rule).

Then we calculate by separate operations the largest possible D_a, D_b, D_c, … and the mean deviations $da, db, dc, \ldots$ and find

$$F_1D_a + F_2D_b + F_3D_c + \ldots \text{ or } F_1da + F_2db + F_3dc + \ldots$$

And now we only have to take the square root of the sum of the squares and arrive at (3.4) and (4.6) and thus get the largest possible error and the mean error of F.

19. A simple example of using this rule. The error of measuring the volume of a prism. It is useful to throw light on these calculations by many examples. We provide sufficient explanation for indicating the application of the rule. First of all we may consider an extremely simple case, the measurement of a rectangular prism. Denote its dimensions by *a, b* and *c*. Its sought volume $x = abc$,

$$dx = bcda + acdb + abdc.$$

Those coefficients measure respectively the influence of the partial errors *da, db, dc* and acquaint us how each partial error contributes to the total error. These contributions are the more essential the more extensive are *bc, ac, ab*. Finally,

$$Dx = \sqrt{b^2c^2Da^2 + a^2c^2Db^2 + a^2b^2Dc^2}. \tag{3.19}$$

20. Definition of the relative error and of the logarithmic differential. We can provide another form for the preceding calculation[27]:

$$x = abc,\ \ln x = \ln a + \ln b + \ln c$$

and, after differentiation,

$$\frac{dx}{x} = \frac{da}{a} + \frac{db}{b} + \frac{dc}{c}. \tag{e.20}$$

Each term on the right side expresses a relative error; for example, the first term is the relative error of *a,* and the left side, by the same definition, is the relative error of *x*. For the present case equation (e.20) tells us that the relative error of the volume is the sum of the relative errors of the three dimensions. This special relation is due to the very simple form of the function *abc* and it does not hold for other functions.

We may write equation (e.20) in the form

$$dx = \frac{x}{a}da + \frac{x}{b}db + \frac{x}{c}dc$$

and, by the principle expressed above, after denoting the boundaries of the largest errors by Dx, Da, Db and Dc,

$$Dx = \sqrt{\frac{x^2}{a^2}Da^2 + \frac{x^2}{b^2}Db^2 + \frac{x^2}{c^2}Dc^2}$$

which is equivalent to the preceding equation (3.19).

21. Concerning the same problem we suppose that the boundaries of the largest relative errors are the same for each of

the three dimensions and find out the boundary of the largest relative error of the volume. We mentioned that the quotient *da/a* expresses the relative error of the measurement of *a*. There is no reason to believe that D*b/b* or D*c/c* differ from D*a/a*. Quantities *a, b, c* are of the same kind measured by the same method so that generally speaking, excepting particular cases in which the methods and/or instruments for measurement are different, these three fractions ought to be supposed identical. Therefore

$$\frac{Da}{a}=\frac{Db}{b}=\frac{Dc}{c}=\ldots,\ \ Dx=\sqrt{\frac{3x^2}{a^2}Da^2}=\frac{x}{a}Da\sqrt{3},\ \ \frac{Dx}{x}=\frac{Da}{a}\sqrt{3}.$$

Now, D*x/x* is the boundary of the relative error of *x*. Therefore, the boundary of the relative error of the volume is the product of the square root of 3 multiplied by the relative error of one single dimension. That result can only be justified by the analytical theory described above.

22. Calculation of a vertical height. The expression of the boundary of the error. Here is another problem almost as simple as the previous whose consequences are still more remarkable. If we measure a horizontal base *b* and the angle α situated in the vertical plane, *b*tanα will be the height *x* which we wish to determine. The main equation is

$$x = b\tan\alpha,\ \ln x = \ln b + \ln\tan\alpha, \tag{1.22}$$

$$\frac{dx}{x}=\frac{db}{b}+\frac{d\alpha}{\sin\alpha\cos\alpha}. \tag{2.22}$$

The equation (2.22) indicates the relative error of the unknown height *x* as the sum of two parts, the relative error of the base *b* and the error $d\alpha$ divided by $\sin\alpha\cos\alpha$. We have

$$dx=\frac{x}{b}db+\frac{x}{\sin\alpha\cos\alpha}d\alpha.$$

The coefficients of *db* and *d*α express the parts of the total error corresponding to *db* and *d*α, i. e. *x/b* and *x*/sinαcosα or tanα and[28] $b/\cos^2\alpha$.

23. The error of the measurement of an angle is not relative at all, but always expressed by an abstract number. From equation (3.19) which shows the boundary of the errors of *x* we have

$$Dx = \sqrt{\tan^2 \alpha Db^2 + \frac{b^2 D\alpha^2}{\cos^2 \alpha}}.$$

The errors of the measurement of an angle are absolute rather than relative because the difference between a given angle and its value provided by the instrument, that is, the error of measurement, is independent from that angle.

24. In the preceding problem the relative error of the unknown height consists of two parts. When we apply that remark to equation (2.22) we see that the relative error of the unknown height is formed by two parts, a relative error and a number. Suppose that the angle α is measured exactly, so that $d\alpha = 0$, then $dx/x = db/b$ which means that the vertical height x is measured as precisely as b. This consequence is evident. If the base is measured absolutely exactly, $db = 0$ and

$$\frac{dx}{x} = \frac{d\alpha}{\sin\alpha\cos\alpha}.$$

However, in the general case neither db nor $d\alpha$ disappear.

25. The boundary of that relative error and the mean relative error. When applying the principles described above and denoting the boundary of the error of x by Dx we have

$$\frac{Dx}{x} = \sqrt{(\frac{Db}{b})^2 + (\frac{D\alpha}{\sin\alpha\cos\alpha})^2} = \sqrt{(\frac{Db}{b})^2 + (\frac{2D\alpha}{\sin 2\alpha})^2}. \qquad (1.25)$$

This is the expression of the largest relative error of the vertical height which is not directly measured but derived by issuing from a measured base and an angle in the vertical plane. To express the mean error it suffices to replace the symbol D in (1.25) by d which denotes the mean error and, according to memoir [i], it concerns the rule which serves to determine that mean error.

26. A remarkable consequence of the previous solution. Such solutions determine the most favourable conditions for precision. Application to the previous problem. Now we describe one of the most useful applications of the principle which serves to estimate the precision of the results of calculation, to find out the most favourable conditions for precision. For example, the previous problem: it was impossible to measure directly the vertical distance x. Therefore, we ought to indicate the most favourable disposition, that is, the value of α that provides the most precise result if no other condition is changed.

No one will fail to know from experience[29] that the best angle is 45°. This problem is so simple that the cause of the mentioned choice is not difficult to perceive, but we need a general method for applying it in more complicated cases. This method is based on the preceding notions and the sought precision is expressed by a number. This suffices to discover the condition for the figure which will *maximize* that expression [that number]. For example, it is ascertained that the boundary of the maximal error or [rather] of its relative error in this case is (1.25).

We see that the left side of (1.25) varies with α. Suppose that the instrument which serves for measuring angles is known and that the method of measuring the base is also determined. If these methods do not change we may essentially vary the precision of the result, i. e. vary D*x/x.* This relative error lessens if D*b* and Dα are not changed but sin2α becomes larger. The maximal value of this last mentioned function is unity. So the most precise determination of the height is achieved at α = 45° and

$$\frac{Dx}{x} = \sqrt{(\frac{Db}{b})^2 + (2D\alpha)^2}.$$

The preceding theory provides a method for comparing the relative precision, suffice it to compare the two values of D*x/x* at α = 45° and at some other value.

We have considered maximal relative errors but the same consequence is applicable to the mean relative errors for which we obtain the same formula with *d* replacing D.

We conclude this memoir by a summary of the mentioned propositions.

27. A summary and various remarks. Many quantities *a, b, c,* … are regarded as known since the value of each is measured by an instrument whose application can be repeated. An unknown quantity *x* is expressed by a certain function of those given and the nature of that function is known. Each datum is subjected to a certain error of measurement which we ought to regard inevitable but which cannot exceed certain boundaries.

Those errors evidently influence the error of *x* and the problem consists in the exact determination of the boundaries of the error of *x* given the boundaries of each known quantity. Until that problem is solved, we can only form an improper idea about the error of *x*.

And so, we ought first of all to determine the boundaries of the values of each of the quantities *a, b, c*, … We determine them by applying the instrument many times and by using the general rule provided in the preceding memoir [i]. This rule consists of deriving the sought boundaries when measuring the same quantity (such as *a*) many times.

First of all we calculate the mean values of those same quantities [Fourier describes the steps leading to the calculation of]

$$g = \sqrt{\frac{2}{m}(a_1^2 + a_2^2 + \ldots + a_m^2) - (\frac{a_1 + a_2 + \ldots + a_m}{m})^2}.$$

Here, a_1, a_2, …, a_m are the observed values of *a*. That number, *g*, allows us to find out the boundary of the error of *a*; take 3*g* and regard it as certain that the exact value of *a* is contained within $a - 3g$ and $a + 3g$.

For ascertaining the exact sense of that proposition we should consider that, after repeating the measurement of *a* infinitely many times and calculating the mean value of those infinitely many measures, the obtained mean will be a completely fixed quantity. Indeed, after measuring that same datum once more infinitely many times, the newly computed mean will not at all differ from its previous value. The mean value of infinitely many results of measuring the same magnitude does not vary, we will always get the same value, call it *A*.

If however that magnitude is only measured a finite number of times the mean value of those measurements will generally differ from the fixed quantity *A*, and it is this difference which we call the *error* of *a*. It is contained within $a - 3g$ and $a + 3g$.

We ought to remark that this consequence is applicable to *random* errors with which the absolute value of *a* can be corrupted. If [in addition] the instrument is corrupted by a constant error which invariably reproduces itself as soon as the instrument is applied, this error obviously persists in the mean[30].

As to random errors, they disappear to an ever greater extent with the increase of the number of operations[31]. It is always possible, when indefinitely increasing that number, to get rid of all random errors: the difference between the mean values and the fixed magnitude *A* then becomes ever smaller and can be made smaller than any given quantity.

By similar calculations we determine the mean error of the measured quantity *a*, but by multiplying *g* by 0.47708 rather than by 3. The product is the *mean error* of *a*. The boundary of the absolute error with which the value of *a* can be affected is 3*g*. The product 3g expresses the largest error which can be attributed to the measured quantity *a*. It is not rigorously impossible but its probability is extremely low, lower than 1/20,000.

After determining the boundary of the error of a, and its mean error we should derive the boundary of the unknown x and its mean error which is a certain function $F(a, b, c, \ldots)$ of the measured quantities. For calculating those two errors we operate in the following way.

1. Denote by Da the boundary of the error of one of the data and by Db, Dc, … the same for the other data. These small quantities Da, Db, Dc, … are indicated by applying a procedure which I am now describing. We substitute the immediate results of the measurements of $a, b, c, \ldots$ in the function $F(a, b, c, \ldots)$ and obtain the first result, F. Then we augment one of those data, say a, by that small quantity Da, by the boundary of the error of a, and calculate $F(a + \mathrm{D}a, b, c, \ldots)$, the second result, and denote it by F_1. It can vary very little from F and we denote $F_1 - F = \mathrm{D}_aF$. We deal with the other data in the same way, find D_bF, D_cF, … and calculate

$$D_x = \sqrt{(D_aF)^2 + (D_bF)^2 + (D_cF)^2 + \ldots}\,. \qquad (27.1)$$

We are assured that the value of x is contained within

$F(a, b, c, \ldots) - \mathrm{D}_x$ and $F(a, b, c, \ldots) + \mathrm{D}_x$.

2. We denote by da the mean error of one of the known quantities and by db, dc, … the mean errors of the rest of them. These mean errors are found by applying the rule described above, by multiplying g by 0.47708. Now vary only one of the data, say a, by a small quantity da, compare this result with $F(a, b, c, \ldots)$ and denote the difference by d_aF. After determining such differences for all the data we calculate [Fourier repeats formula (27.1) but replaces symbols D by d]. This is the mean error of x.

We can also determine the small increments D_aF, D_bF, D_cF, … and d_aF, d_bF, d_cF, … by differentiating the given function $F(a, b, c, \ldots)$ with respect to $a, b, c, \ldots$, but the practical operation which I indicated makes it unnecessary.

When excluding very simple cases in which differentiation does not require much work, we find that the usual rule indicates these values D_aF, d_aF, … much easier. I also say that when calculating F, we should vary any one of its arguments, $a; b, c, \ldots$, one at a time, by a very small number Da (when varying a) expressed by a simple number, for example, by 1 minute if a is an angle or a millimetre if a is a length. Then we multiply this correction by the calculated Da. The same is repeated for all other corrections Db, Dc, ... db, dc, … After that it is easy to get D_aF, D_bF, … which enter in the square root of Dx or dx. This numerical calculation is much easier than the dealing with differentiation which is almost always complicated by trigonometric formulas.

The preceding rules determine

1. The boundary of the error of x which is a given function $F(a, b. c. \ldots)$.

2. The mean error of that same unknown.

These two results complete the knowledge obtained by calculation and provide a true idea of the errors to which each application is subjected. The analytic expression of the boundary of the errors of the unknown or of its mean error leads to another remarkable consequence: it indicates how the measured quantities are combined when determining Dx or dx. Therefore, they can solve this problem:

Which conditions for the figure and, in general, which values of a, b, c, … are most favourable for the precision of the results of calculation?

These are the values which lead to the least possible boundary Dx and therefore the mean error dx. For the trigonometric operations, when deriving certain quantities which cannot be directly measured by issuing from measurable quantities, it is therefore important to know which regulated conditions render more precise results. It is easy to distinguish them when the trigonometric expression is very simple (for example, when calculating the vertical distance), but in a bit more complicated cases such a discussion requires lengthier investigations. Nevertheless, the regular dealings are based on theorems which are provided here.

Not only we come to know the preferable figure to which we ought to approach as nearly as possible, we also distinguish which quantities require to be measured more precisely. This theory allows us to estimate the degree of precision and numerically compare the results of certain conditions imposed on the figure with those which exist under different conditions.

In memoir [i] we provided a general and easy rule for estimating the degree of precision of the mean results. Now, we extended the application of that rule to all cases of calculating an unknown value which depended on measurable quantities corrupted by inevitable errors. It follows that that calculated value itself is also subjected to the ensuing error whose boundaries we have determined.

The application of calculations can be therefore compared to the use of an instrument whose precision is exactly known. We think that the publication of these theorems about the errors of measurement and the precision of the results of calculation contributes to the perfection of the application of mathematical sciences. These considerations naturally belong to a collection which aims to observe and state all the principal elements of public prosperity.

Summary of the articles

[Here, Fourier collected the names of the sections or articles of this memoir.]

Notes to memoirs [i] and [ii]

The Notes which precede memoir [ii] concern both memoirs.

Notes to memoir [i]

1. I only found there a tiny passage about the Cheops pyramid (§ 172, wrongly mentioned in the Table of Contents). It ends however with a statement that Fourier was the first to know that the error of mean values increases as the square root of the number of the pertinent particular values.

2. This statement is too elementary and even wrong. I only mention two points. First, the sample of a few thousand observations should be representative. Second, special populations should not be included (and, if needed, studied separately). Antoine Déparcieux the Elder (1705 – 1768) compiled tables of the duration of life *based on religious orders* (apparently, on monks and nuns in the first place), see Pearson (1978, p. 199), and he also thought that annuitants were likely to live longer. Corbaux (1833, pp. 170 – 172) separated mortality tables into five parts according to the conditions of life of the various strata of the population. His idea was worthy, but his tables were obviously imagined: he had not cited any sources and did not explain how he managed to separate the mortality table.

3. *Mathematical analysis of our time* means that Fourier had likely thought about Laplace (Sheynin 2017, § 7.1-5).

4. Analysis can prove this result in the stochastic sense by presuming a normal distribution. However, the property of random errors is rather justified by the experience of practically each observer. The *largest* (which Fourier repeatedly mentioned) is superfluous.

5. Consider, indeed, Quetelet (1848, p. 169):

In Bavaria, it was attempted to prevent impulsive marriages […] *and then the numbers of babies born in and out of wedlock became almost the same*.

When describing the duration of generations (below) Fourier only thought about the male duration but he remarked about the need to consider the female duration as well. It is opportune to mention that separate mortality tables for either sex *only recently* began to appear (Quetelet & Smits 1832, p. 33). Corbeaux (1833) also provided such tables, but see Note 2.

6. Fourier had not explained the sudden appearance of Jakob Bernoulli and later scientists He apparently thought about the Bernoulli law of large numbers. However, for many decades statisticians had not applied it (Sheynin 2017, § 10.7-8).

7. In § 5 Fourier stated that the separation ought to be random which will possibly reveal any changes in the underlying causes. I say: reveal causes in general for which the observations ought to be separated into those describing populations possibly possessing some peculiar trait and all the rest people. In this way Snow (1855) studied cholera. He separated the population of London into those who had been drinking purified water and the rest citizens. Mortality from cholera was about eight times higher in the latter group.

8. The data are so unusual that the explanation of their choice was necessary, but impossible! For one thing, thousands of observations cannot obey one and the same law. In 1828 Quetelet (Sheynin 1986, p. 309) borrowed that example without comment. It was perhaps Fourier who introduced the measure *g* which had been in use at least a few decades.

9. This is not the general case and the chances which determine the theoretical probability could have been unequally possible. This would have required their weighting.

10. Fourier had not even hinted that in great many cases the theoretical probability just does not exist. Statistical probabilities have to be used.

11. This is a property of random errors. See Note 4. And instrumental errors are not the only source of random errors of observation whereas very imperfect instruments cannot be trusted at all.

12. The mean (moyenne) error mentioned many times in the sequel is actually the probable error formally introduced by Bessel (1816, pp. 141 – 142). Many authors including Poincaré (1896) had been unfortunately confused in the same way.

13. Precision cannot be increased indefinitely: unavoidable dependence between observations will prevent it. Another circumstance: systematic errors will become ever more pronounced. Bayes actually said so in his posthumously published note written ca. 1757 (Dale 2003, p. 285). It is however more proper to say that systematic errors determine the *accuracy* of the observations rather than their precision. Later Fourier himself [1829, § 27] noted that precision only concerned random errors.

14. The calculation of superfluous digits had been the general practice of authors up to the mid-20th century. Gauss calculated his measured angles to within 0.″001 (actual precision about 700 hundred times lower); Karl Pearson kept to the same habit, even if not to the same extent, and at least one similar example pertained to Fisher, see a discussion of this subject in *Science*, vol. 84, 1936, pp. 289 – 290, 437, 483 – 484 and 574 – 575.

In his previous statement Fourier mentioned an interval in an unusual way (in the inverse order of its ends). The same comment applies to many instances in his second memoir.

Concerning the lines just below: the boundaries of errors tend to widen with the number of observations. Just after W. Jordan introduced the three-sigma (3σ) test Helmert (1877) investigated this circumstance which is also applicable to Fourier. See Harter (1977, date of Preface, p. 63) and Sheynin (1995, pp. 79 – 80).

15. This statement heuristically resembles Gauss' opinion (letter of 1839, *Werke*, Bd. 8, pp. 146 – 147): maximal probability of a result is less important than a least disadvantageous game of chance. That was his main reason for abandoning his first justification of least squares of 1809.

16. Fourier obviously had in mind his paper (1819).

Notes to memoir [ii]

17. Fourier many times equates known and measured (and corrupted by unavoidable errors) quantities.

18. To remind: Fourier denoted memoir [i] by letter M.

19. The *largest* is not needed at all, but it occurs time and time again.

20. This passage (and a few others) clearly shows that Fourier was not acquainted with field geodetic work. Indeed, **1)** No one ever measured bases more than twice (if the measurements did not differ too much). **2)** Fourier described the determination of some quantity by intersection, but he obviously did not know this French (and English) term. **3)** See Note 12.

21. Unusual mention of intervals: see Note 14.

22. That term was barely applied by Fourier.

23. Here is a prime example of unnecessary repetitions (omitted in the translation): … *when applying formula* (1) ... *each time assuming formula* (1).

24. I utterly fail to understand Fourier! Instead of proving algebraically by a single line that $(a + b + \ldots)^2 > (a^2 + b^2 + \ldots)^{1/2}$, he proves it geometrically in a somewhat complicated way.

25. Fourier joined the army as a scientific adviser during the French campaign in Egypt and Syria (1798 – 1801).

26. Why this dated term, analysis of chances?

27. Notation: $\log_e n = \ln n$.

28. Fourier unnecessarily repeated so much, but had not explained the derivation of the following expression. I have not checked his formula from § 23.

29. A strange statement.

30. Fourier only mentioned the simplest case of a systematic error. Cf. Note 13.

31. An unfortunate expression which Fourier specified in his next statement.

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III

Oscar Sheynin

On the history of the statistical method in natural science

This is a summary of my contributions Sheynin (1980, 1982, 1984a, 1984b, 1985). In § 1 I discuss some preliminary considerations and formulate my main conclusions which follow from §§ 2 – 6.
I do not dwell on the history of the theory of errors (of the application of the statistical method to the treatment of observations and measurements). Its relations with statistics (see a bit below) are complicated by the notion of true value (Sheynin 2007). Much material on the history of probability, and inevitably, of statistics, is in Sheynin (1974; 1977).

1.1. The main definitions. Kolmogorov (1948, p. 216) thought that statistics is usually understood as a science which studies sociology. Kruskal (1978, p. 1072), however, decided that

Theoretical statistics is a formal study of the process leading from observations to inference, decision etc. which usually proceeds in mathematical terms.

Usually means that general natural scientific methods can be involved, see also § 1.2. Mathematical statistics is less extensive than theoretical statistics: unlike the latter, it does not study either the collection or preliminary investigation of data. I (1998) attempted to study the evolution of the concepts of probability and statistics, but anyway mathematical statistics really originated with Fisher and Student (Gossett).

Tukey (1972) stated that statistical theory was not identical with mathematical statistics (certainly not) and quoted an apparently emotionally motivated Anscombe (1967, p. 3n): [the notion of] *mathematical statistics is a grotesque phenomenon*.

And here is the definition of mathematical statistics by Kolmogorov & Prokhorov (1974/1977, p. 721):

<u>Statistics, mathematical</u>*, the branch of mathematics devoted to the mathematical methods for the systematization, analysis, and use of statistical data for the drawing of scientific and practical inferences. Here, the term* <u>statistical data</u> *denotes information about the number of objects in some more or less general set which possess certain attributes*.

They also define *statistical method*: the method of investigation based on the consideration of statistical data.

1.2. The content of statistical studies. Each such study is naturally separated into three steps: observations, their preliminary study, justification and formulation of inferences. During the latest century the volume of information had increased so much that the significance of its preliminary study became even more essential. This work means compiling summaries, drawing diagrams, and revealing structures and anomalies in the data by means of simple mathematical (especially statistical mathematical) and natural scientific methods. Even in the beginning of that, the 19th century, complaints were voiced about the great increase in data (Lüder). I only refer to Biot (1855).

The main initial progress in the initial study of data was due to Tukey (1972 and other contributions), see Andrews (1978).In population statistics, Quetelet (1846, pp. 281 – 305) described what amounted to the prehistory of this problem. The third step sometimes falls out, as when compiling statistical tables. This direction of statistical studies became named tabular statistics, and it has its own history which properly began with Anchersen (1741). Two Russian authors ought to be mentioned as well: Kirillov (1831, 1977) who compiled his manuscript in 1727, and Golitzin (1807) whose paper I have not seen. Fourier (1821 – 1829) should be also named.

I separate the history of statistical research (or method) into three stages. At first, conclusions were being based on (statistically) noticed qualitative regularities, a practice which conformed to the qualitative essence of ancient science. Here, for example, is the statement of the Roman scholar Celsus (1935, p. 19):

Careful men noted what generally answered the better, and then began to prescribe the same for their patients. Thus sprang up the Art of medicine.

The second stage (Tycho in astronomy, Graunt in demography and medical statistics) was distinguished by the availability of statistical data. Scientists had then been arriving at important conclusions either by means of simple stochastic ideas and methods or even directly, as before. During the present stage, which dates back to the end of the 19th century, inferences are being checked by quantitative stochastic rules.

And here are my conclusions to which I refer by their number shown in curly brackets: {1}, {2}, etc.

1. In each branch of natural science the statistical method had been developing independently. It is opportune to mention in addition that Laplace had resolutely transferred probability from pure to applied mathematics and that, in spite of the work of Chebyshev and his eminent students, the return of probability to its previous field only occurred in the 1920s or 1930s.

2. There had existed an unavoidable and incessant contradiction between statistics and the concrete science to which it was applied. Statistical data and conclusions showed the direction ahead for the science, but after some of its progress the data and conclusions became useless. To supplement this statement I say that in the 19th century some conclusions from statistical data had either been forgotten or rejected. It is possibly worthwhile, if only from the methodical viewpoint, to repeat the pertinent calculations.

3. By the mid-19th century statistical totalities had been recognized in some branches of natural science.

4. In the 19th century there appeared branches of natural science which were directly linked with statistics whereas statistics even determined the development of some other branches.

5. In many branches of natural science most important results had been achieved simply by looking at the results of statistical observations.

6. Tabular statistics became applied as well.

7. In some branches of natural science the statistical method had been initially restricted to the derivation of mean values, but later it had included the study of the derivations from these values, then laws of distribution.

8. The Darwinian evolution of species can be described as a discrete stochastic process.

9. Humboldt's theoretical merits in natural science are explained by his studies of the mean values (conditions) of natural events.

10. The first numerical study of correlational dependences was due to Seidel.

11. Clausius introduced a linear function of an integral law of probability. It was infinitely divisible.

12. Maxwell forestalled the Poincaré idea that unstable equilibrium in which slight causes lead to essential effects characterizes a random event.

13. The prehistory of the preliminary study of statistical data (§ 1.2) should include the work of Halley (Chapman 1941, p. 5), i. e., the introduction of contour lines into science, Humboldt and Galton.

2. Medicine

The statistical method was introduced into medicine along several directions. Population statistics, however, should have been connected with statistics from the time of its appearance, but was not closely linked with any embryo of medical statistics. True, Graunt was also meritorious for his study of the influence of various diseases on mortality, and much later Süssmilch (1758) emphasized that poverty and ignorance were conducive to the spread of epidemic diseases. He also rendered an essential service to statistics by a general study of population statistics (1741), see also Birg (1986).

The main cause of that unfortunate fact was the ignorance of statisticians, their fear of probability. Even a most eminent statistician, Knapp (1872, p. 115), called probability difficult and hardly useful beyond the sphere of games of chance and insurance.

It was Jakob Bernoulli (1713) who paved the way for justifying statistical probability by proving that, after a long series of *Bernoulli trials*, it became not worse than theoretical probability. His inspiration was essentially due to Arnauld et al (1662) whose merits are not sufficiently described.

Nevertheless epidemiology and public hygiene (the forerunner of ecology) which originated in the mid-19th century (§§ 2.4 and 2.5) essentially needed statistical data. Surgery during that same period also regarded statistics as a necessary means of investigation (§ 2.3) {4}.

2.1. The numerical method {6}. It was introduced into medicine by Louis (1825) and had remained in vogue for a few decades whereas Pirogov (1849a, p. 5) and then Davidov (1854, p. 84), in spite of his understanding its limitations, positively characterized it whereas Greenwood (1936, p. 139) excessively praised it more than a century after its appearance.

This method consisted in studying and compiling statistical summaries almost without any application of stochastic considerations. And even before Louis physicians and scientists in general (D'Alembert 1759; Black 1788, Condorcet, 1795) argued that statistical data should be used in medicine in the manner of the future numerical method. Pinel (1801) published a statistical study of the treatment of mental patients in the manner of the numerical method. True, at that time his treatment had been necessarily reduced to the organization of a humane regime. Neither he, nor Louis himself thought about the estimation of the reliability of their conclusions.

2.2. Elements of mathematical statistics. In 1835 the Paris Academy of Sciences discussed the application of the theory of probability (and, actually, statistics) to therapeutics but did not recommend anything definite although Double et al (1835) stated that, concerning the discussed problem, medicine did not differ from other sciences. That same Double (1837), however, expressed himself quite differently!

Gavarret, a former student of Poisson, took to medicine and published a book (1840) on the principles of (the yet non-existing) medical statistics. He followed Poisson, as he indicated, and recommended physicians to check the reliability of their conclusions by the De Moivre – Laplace limit theorem (which presupposed a large number of medical observations) according to the chosen [significance level] and to introduce and apply [null hypotheses].

For many decades the Gavarret formulas had been copied from treatise to treatise and at least fostered the future introduction of mathematical statistical ideas and methods. Ondar (1971) described Davidov's pertinent merits.

It was difficult to collect many observations. Here is the opinion of a noted physician (Liebermeister ca. 1877, pp. 935 – 940): physicians have applied the theory of probability so seldom *mainly since its analytical arsenal was too imperfect and awkward.* It is impossible to collect *thousands* of observations (as required by limit theorems; an obvious exaggeration). Gavarret and Poisson stated that 0.9953 *is a sufficient measure of probability*, but if *the successes of two methods of treatment are only as 10:1, would not that be sufficient for preferring the first one*?

His criticism was still valid. Modern statistics including the theory of errors cannot restrict its activity to the case of a large number of observations. Therefore, Liebermeister rather than Gavarret was the pioneer of medical statistics.

2.3. Surgery. In this branch of medicine the statistical method began to be applied in 1839. The introduction of anaesthesia (which sometimes was accompanied by serious side effects) required the comparison of mortality from amputations performed with and without that novelty. J. Y. Simpson (1847 – 1848) published such a study, then Pirogov, who introduced anaesthesia into military surgery, compiled another one (1849b). Simpson deliberately issued from heterogeneous data, he mistakenly believed that his choice ensured higher reliability for his conclusions, cf. § 5.4. Anyway, the definitive introduction of anaesthesia invalidated all the previous data, and the same followed from the introduction of the Listerian method (Lister 1870, pp. 123 – 136) {2} whose beneficial effect became evident at once {5}.

The International Statistical Congress (*Congrès* 1860, published 1861, p. 247) agreed with the status of surgical hospitals as investigated by Florence Nightingale (1859/1863, p. 159). She indicated that post-operational complications reflect an unsatisfactory condition of a hospital better than high mortality of the patients in general. She, as also Pirogov (1849a, p. 19), Virchow (1868 – 1869/1879, Bd. 2, pp. 6 – 22) and Simpson turned attention to the higher mortality in large hospitals. According to Simpson (1847, pp. 318, 319, 325 – 326; 1869 – 1870/1871, p. 399) mortality steadily increased with the number of beds (actually, with the worsening of hygienic conditions of the patients). A steady and practically real change of indicators became a convincing argument of medical statistics {5}.

Due to the unreliability of statistical data in military surgery, Pirogov (1865 – 1866, vol. 5, p. 20) recommended to believe only *sensible* observations. After comparing the mortality peculiar to the conservative treatment and operations he (1864, p. 690) called his time *transitional*. In the first decades of the 19th century, as he noted, the *previous school* had not been considering the danger of operations whereas now that school *is shaken by statistics*. However, new principles will not be established until the appearance of statistics of high quality. Pirogov was not interested in mathematical statistics, but he extensively applied the statistical method and recognized the existence of stable mass phenomena. Thus, he maintained, that the abilities of physicians only led to *barely noticeable* fluctuations of the the results of healing a definite illness.

This viewpoint allowed him to pay utmost attention to the *organization* of military surgery and even military medicine as a whole {5}.

2.4. Epidemiology. Modern epidemiology tries to predict the course if epidemics. No such aim had been set in the previous times; Farr, in 1866 (see Brownlee 1915) was an exception, but his study belonged to the veterinary science rather than to medicine.

Inoculation of smallpox from an ill to a healthy person led to the appearance of statistical problems. Daniel Bernoulli devoted his classical investigation (1766) to their solution, as did D'Alembert (1761; 1768) although the such studies had first appeared in the 1720s. And still the involved statistical problems had not been properly solved (Karn 1931, p. 290).

A new and final stage in the struggle against smallpox began with Jenner. The results of the new inoculation, or vaccination, almost at once showed the exceptional benefit of that preventive measure but statistical problems appeared once more. A simplest example concerned the expiry period of the vaccine. Simon (1887) concluded that the proper estimation of vaccination was only possible on the basis of a national statistics of the population.

In the 19th century (and earlier) cholera epidemics had repeatedly devastated Europe. Pettenkofer (1886 – 1887) published a monstrous survey of materials on cholera with a large number of tables, but he was unable to interpret them duly. Snow (1855, p. 86) compared the mortality from cholera of the London population which had been drinking purified or impure water. The purification (even in a patently insufficient manner) lessened mortality from cholera about eight times which was sufficient for the obvious conclusion {5}.

But still Pettenkofer (1865, p. 329) believed that an epidemic in a given locality was impossible without an appropriate predisposition (threshold value) and his opinion has been more or less upheld.

The International Statistical Congress repeatedly discussed the statistics of epidemic diseases, especially cholera. In 1872 (*Congrès* 1872, published in 1872 – 1874, t. 1, p. 45), it recommended a statistical check of the Pettenkofer statement but this had not been done.

Seidel (1865 – 1866) investigated the dependence of the monthly cases of typhoid fever on the level of subsoil water, and then on both that level and the rainfall. It occurred that the signs of the deviations of these figures from their mean yearly values coincided twice more often than not and Seidel quantitatively (although indirectly and with loss of information) estimated the significance of the studied connections. His work remained, however, completely forgotten and Weiling (1975) was likely the first to recall it. On a lower level Soyka (1887) and Winslow (1943/1967, p. 330) continued his work.

2.5. Public hygiene. From its origin in the mid-19th century, public hygiene began statistically studying a large number of problems, especially those caused by the Industrial Revolution in England and, in particular, by the great infant mortality. Thus, in Liverpool only 2/3 of the children of gentry and professional persons lived to the age of five years (Chadwick 1842/1965, p. 228).

Pettenkofer (1873) estimated the financial loss of the population of Munich ensuing from such diseases as typhoid fever and his booklet can be attributed to this discipline. In Russia his student Erismann (1885) published a contribution on sanitary statistics. I refer to its Russian edition of 1887. The medical profession, as he stated (vol. 1, p. 7), had now a better understanding of the role of statistics, but (vol. 2, p. 3 of Supplement) *until recently famous practitioners did not recognize medical statistics*.

Mortality in the army and in prisons had also been studied. The extremely successful student of the former was certainly Florence Nightingale, but I leave this theme alone. Farr (paper of ca. 1857/1885, p. 148) left a general conclusion which he chiefly explained by the horrible state of public hygiene:

Any deaths in a people exceeding 17 in a 1,000 annually are unnatural deaths. If the people were shot, drowned, burnt, poisoned by strychnine, their deaths would not be more unnatural than the deaths wrought clandestinely by disease in excess of […] *seventeen in 1,000 living*.

3. Biology

3.1. Various problems in the pre-Darwinian period. Not later than in the mid-18th century (Adanson 1757) botanists began to study statistical methods, to discuss *natural* methods of classification of plants, – such which preserve the *distances* between species as much later recommended Aug. P. Decandolle (1813/1819, p. 29). This additional requirement was linked with the notion of multivariate statistics. Also see Adanson (1763, p. cccxiv).

Réaumur (1738, pp. 558 – 559) proposed a law of the sum of temperatures: leaves, then flowers and fruits, of a given species of plants, appear after the sum of the mean daily temperatures reaches a certain value. Aug. P. Decandolle (1832, t. 1, pp. 432 – 434) qualitatively compared the results of observations (which he recommended to standardize) with that law. However, Quetelet (1846, p. 242) proposed a law of the sum of the squares of the temperature but was unable to compare quantitatively both laws.

In t. 2 of the cited contribution Aug. P. Decandolle published extensive statistical data on the consumption of oxygen by plants in darkness (p. 550), on the content of water and sugar in fruits (pp. 584 – 585) etc. Babbage (not later than 1833) began to propagandize the statistical study of animal life (1864, p. 376) where, on pp. 295 – 299 he published his questionnaire. Earlier, he published his tables (1857).

Baer and associates (1860 – 1875) published a practically important study of the fisheries in Russia. His main associate was an eminent statistian N. Ya. Danilevsky. Valt (1978, pp. 107 and 110) suggested that this, and another research made by Baer in 1852 directed him towards theoretical problems in animal ecology.

Pasteur (1882) tested the effect of his vaccine against anthrax on many thousands of animals. The results were brilliant and there was no need in mathematically confirming them {5}.

Compilation of botanical statistical data was a necessary component of geography of plants, of a discipline created by Humboldt &

Bonpland (1815) and Humboldt (1816) {4} although Alph. Decandolle (1855, t. 1, p. vi) also named Linné, his own father Aug. P. Decandolle and Brown. Darwin (letter of 1881; 1903, vol. 2, p. 26) only named Humboldt. The International Statistical Congres (*Congrès* 1858) published a questionnaire partly devoted to the geography of plants and zoogeography.

Anthropometry which originated in the second half of the 19th century {4} was also directly linked with statistics. Quetelet (1871) was its pioneer, and the term itself, as he stated, was due to Humboldt's advice.

3.2. Various problems: Darwin. Darwin had to apply statistics for solving many problems and his contributions carry a large number of statistical tables; he used the statistical method both manifestly and indirectly. Here are some examples.

1. When studying a rare deviation in man (polydactyly) Darwin (1868/1885, vol. 1, p. 449) asked Stokes to calculate the probability of its occurrence. Stokes solved this problem apparently by applying the all but forgotten Poisson distribution. See also Maupertuis (1756b/1756a, t. 2, p. 286).

2. Darwin (1876/1878, p. 15) decided that cross-fertilization of plants should be preferred over their spontaneous pollination but asked Galton to investigate this subject. Galton (Ibidem) confirmed his opinion by comparing the sums of the heights of the seedlings and even by their ordered heights (by order statistics).

Darwin's requests for help were praiseworthy.

3. Darwin (1881/1945, pp. 52 – 55) studied how earthworms had been dragging small objects (small paper isosceles triangles) into their burrows. It occurred that the earthworms preferred to drag the triangles by seizing some places rather than randomly. He checked several versions of random dragging and in this respect he forestalled the celebrated problem of the length of a random chord of a given circle (Bertrand 1888; see Sheynin 2003).

3.3. Evolution of species: Darwin (1859). I reconstruct now Darwin's model of evolution. Introduce an n-dimensional (possibly with $n = \infty$) system of coordinates, the body parameters of individuals belonging to a given species (males and females should, however, be treated separately), and the appropriate Euclidean space with the usual definition of distances between its points. At moment t_m each individual (separated from others by *horizontal* variations) is some point of that space and the same takes place at moment t_{m+1} for the individuals of the next generation. Because of the *vertical* variations, these, however, will occupy somewhat different positions. Introduce in addition point (or subspace) V, corresponding to the optimal conditions for the existence of the species, then its evolution will be represented by a discrete stochastic process of the approximation of the individuals to *V* (which, however, moves in accordance with the changes in the external world) and the set of individuals of a given generation constitutes the appropriate realization of the process. Probabilities describing the process (as well as estimates of the influence of habits, instincts, etc.) are required for the sake of definiteness, but they are of course lacking.

The main mathematical argument against Darwin's hypothesis was that a purposeful evolution under "uniform" randomness was impossible; the notion of randomness in general was barely known. Only Mendel's contributions (1866; 1866 – 1873/1905), forgotten until the beginning of the 20th century, answered such criticisms. True, great many objections and problems still remained, but at the very least Darwin had transformed biology as a science. In addition, his work was responsible for the appearance of the Biometric school {4}. Pearson, its head, paved the way for mathematical statistics to become later an independent discipline. The stochastic nature of the hypothesis of evolution was evident both to its proponents and opponents, only Boltzmann (§ 6.4) somehow thought that it was mechanical.

An essentially new stage in the development of the Darwinian ideas had occurred at the end of the 19th century (Mendel, see above). Then the newest stage began (De Vries 1905) and somewhat later (Johannsen 1922/1929). De Vries stressed the importance of *sports* (although did not explain their relation to mutations, a term that he himself introduced somewhat earlier) which considerably strengthened the theory of evolution.

Andersson (1929), who briefly discussed the work of Johannsen, quoted him, regrettably without indicating the source:

The science of evolution has turned into an Augeas stables which really ought to be mucked.

And here is Johannsen himself.

p. 355. *Galton's statistics of heredity were quite erroneous – a combination between collective measurements of unsorted rough material and biological analysis of the real units of certain populations*.

p. 356. *These statistical researches in heredity are naturally of importance from the sociological point of view and of practical interest in insurance calculations and so on. But they do not reach the biological problems of heredity*.

p. 357. *The* [genotypic] *differences are <u>discontinuous</u>* […] *rather contrary to Darwinism*.

p. 359. *We cannot do without statistics!*

3.4. Statements made by biologists and other scientists. I adduce these statements naming, in a generalized and sometimes formalized manner, their subject and explaining its essence, but I certainly omit Darwin. These statements mostly concern the evolution of species which began to interest biologists since mid-18th century and have to do with variations which perhaps gradually became a main object of biological research {3}.

1. Adanson(1757, p. 61; 1763), Aug. P. Decandolle (1813/1819, p. 29). Natural classification of plants. *Organisms are points in a many-dimensional space*

2. Humboldt (1845, p. 82). Mean conditions (states) in nature. *Idea of random horizontal variation*

3. Maupertuis (1745/1756a, p. 120 – 121). *Vertical variations are random and small*

4. Maupertuis (1751, p. 148*), Cournot (1851, p. 119). Randomness. *Its role in evolution is restricted*

5. Lamarck (1809, pt. 1, chapter 7; 1817, p. 450); Maupertuis (1756b/1756a, p. 276). Change of external conditions. *Horizontal variations are random and can lead to changes in species*

6. Lamarck (an VIII (1800)/1906, p. 465; an X (1802)/1906, p. 511; 1809, t. 1, Chapter 7). Evolution of species. *Is a universal phenomenon*

7. E. Geoffroy Saint-Hilaire (1818 – 1822, 1822, p. 121). New species. *They originate due to random mutations*

8. Goethe (1790/1891, p. 120). Species of plants. *They change* (bilden)

9. Comte (1830 – 1842, t. 3/1893, No. 40, pp. 234 and 278, No. 42, pp. 444 and 446). Evolution of species. *Due to external changes*

10. I. Geoffroy Saint-Hilaire (1859, title of the pertinent section). Species and their evolution. *Species vary restrictively*

4. Meteorology

4.1. Stages of development. Buys Ballot (1850, p. 629) separated the history of meteorology since the 19th century into three stages: the study of the mean conditions or states (Humboldt); of the deviations from those states (Dove); and prediction of *meteorological events* {7}. The first two stages had obviously been of a statistical nature {4}, the third one began in the 1870s with the coordination of meteorological observations carried out in different countries and the use of weather charts. This novelty allowed to study the temporal, if not spatial/temporal distribution of the elements of the weather and to begin really predicting some meteorological phenomena. The trust in general conclusions made by issuing from observations at isolated stations was undermined and in particular the conviction that the Moon influences the weather had disappeared, see § 4.2 {2}.

Köppen (1875, pp. 260 – 261) indicated that weather forecasts *have gained a considerable degree of probability* ever since it was discovered that the direction and force of the wind depend on the distribution of atmospheric pressure.

4.2. The influence of the Moon. It had been studied since the beginning of the 18th century. Toaldo (1777) collected the data for 1671 – 1772 about the preservation and change of the weather in various regions of the Earth as depending on the different phases of the Moon and maintained that that influence was essential. He could have, but did not check his conclusion by the De Moivre limit theorem. Anyway, Toaldo certainly applied heterogeneous data whereas the binomial pattern barely suited his deliberations (§ 4.7) and *change of weather* was not definitive enough.

Lamarck was of the same opinion. He (1800 – 1811, No. 6, p. 13; No. 11, pp. 23, 520) isolated 23,520 (!) *genres* of the mutual positions of the Moon, the Sun and the Earth which affected the weather in one or another way but had not even hinted at a quantitative check of his deductions.

Schübler (1830, p. 78) suggested that the lunar influence was perhaps mostly occasioned by *chemical changes* in particles in the air. Very soon
Arago (1858b, p. 25) noted that, unlike the general population, scientists did not [anymore] believe in that influence, but he himself definitively sided with the scientists only in that year (1858a, p. 1). Both references are to his *Oeuvres Complètes* which had not provided the dates of the original publications; I found them: 1832 and 1845 respectively.

Muncke (1837, p. 2072) held the opposite opinion. He studied this problem on pp. 2052 – 2076 of his authoritative review. Glaisher (1867, p. 378; 1869, p. 347) sided with him after studying the direction of the wind and rainfall at Greenwich, but on p. 350 of his second paper he noted that his opinion still ought to be checked. And then Köppen (1873, p. 241) simply declared that the influence of the Moon was insignificant.

It is barely noticed that Lambert had studied that same influence and that Daniel Bernoulli encouraged him to publish his future results, if duly justified, irrespective of their conclusion (Radelet de Grave et al 1979, p. 62). And Lambert did publish his conclusion (1773) which I have not seen.

4.3. Organisation of observations. In the 1730s and 1740s regular observations had been carried out in several Siberian cities and in 1733 Bernoulli compiled directions for their work (Tikhomirov 1932). Nets of meteorological stations had appeared in the second half of the 19th century. The Societas Meteorologica Palatina (Mannheim) was established in 1780 and it existed for about 20 years and its stations were situated in several countries and worked according to a single set of rules (Tikhomirov 1931). A few decades earlier similar work had been initiated by the Paris Société Royal de Médecine, although perhaps on a lesser scale (Kington 1970). Meteorology was the first science to coordinate field observations on an international level.

Lamarck (1802) stressed the need to maintain a net of stations working according to a single plan for each large country (France!).

Observations in four French cities had indeed begun and he himself participated in that activity in Paris (Ibidem, pp. 60 – 65).

Belgian observations began in the mid-19th century under Quetelet. Köppen (1875, p. 256) noted that a network of Belgian stations had been observing

Ever since the early 1840s and [their observations] *proved to be the most lasting and extremely valuable*.

Over the years, Faraday (1991 – 2008) several times expressed his high opinion about Quetelet's measurements of atmospheric electricity, and I especially note two of his letters (to Quetelet, No. 1367 of 1841 in vol. 3, 1996, p. 42, and to Richard Taylor, the then Editor of the *Lond., Edinb. and Dublin Phil. Mag.*, see No. 2263 of 1850, 1999, p. 270).

Not later than in 1843 Humboldt and A. J. Kupffer proposed to the Petersburg Academy of Sciences an organization of observations on a national scale in Russia. Kupffer thought about it in 1829 and Humboldt came to think about it even earlier (*Perepiska* 1962, pp. 94 – 95).

Uniformity of observations was a main problem discussed at the first International Meteorological Congress in 1873.

4.4. The mean states. Humboldt (1845 – 1858, t. 3, p. 288) mentioned *the only decisive method of mean values*. Much earlier he (1818, p. 179) attempted to discover

The mean movements of the atmosphere so as to *distinguish certain types in the succession of phenomena*.

Humboldt (1817) introduced isotherms (p. 466), isotherms of winter and summer (p. 532), showed the isotherms of 0, 5, 10 and 15° on a map of the world (p. 502) and evaluated the fall of the temperature with altitude (p. 594). He thus separated climatology from meteorology.

Dove (1837) maintained that the deviations from mean states ought to be studied as well. For his part, Humboldt (1818, pp. 179 and 190) stated that it was difficult to separate the influence of the *causes perturbatrices* which explained the slow progress of meteorology.

Dove introduced monthly isotherms and indicated (1848, p. 345) that the yearly isotherms ought to be explained by them. He (1839, p. 285) also formulated the aims of meteorology: determination of middle values; derivation of the laws of periodic changes; and discovery of the rules for finding out the irregular changes.

Monthly isotherms describe the temporal distribution of the temperature, but Dove (1848, p. 401) also indicated that there exist spatial deviations so he thus came near to the study of spatial – temporal distribution of the temperature.

Köppen (1874, p. 3) agreed that the introduction of the arithmetic mean into meteorology was a most important step, but that its rule precludes the discovery of causal dependences. In two contributions he (1913; 1936) recommended to introduce elements of the correlation analysis.

Lamont (1867, p. 247) noted that the irregular changes in the atmosphere are not [random variables in the stochastic] sense and (p. 245) recommended to study the deviations of observations made at the same time at differing stations. Quetelet (1849 – 1857, t. 1, Chapter 4, p. 53) hinted at a similar idea, but Lamont (ca. 1837) even claimed that such studies of air pressure and temperature during a year are as reliable as 30 years of usual observations. I note however that temporal changes (say, of the number of sunspots) are nowadays considered as time (statistical) series.

4.6. The dissatisfaction with the error theory. Quetelet (1846, pp. 412 – 414) published the letters which he received from Bravais in 1845 with examples from biology, astronomy and meteorology. They proved that the density curves of observations were often asymmetrical. And still, Quetelet (1853, pp. 63 – 68) returned to the traditional notions and remarked that only *causes spéciales* and anomalies corrupted the [normal] distribution. This later statement had not however prevent him to publish, after some sixteen years, asymmetric curves of tendencies to crime. This goes to show his happy-go-lucky attitude to his statements.

Meyer (1891) studied the treatment of meteorological data and (p. 32) argued that in meteorology the theory of errors (Fehlerrechnung) was inadmissible since the densities were asymmetric. Pearson (1898) applied Meyer's data on cloudiness to check whether his theory of asymmetric densities could be applied for studies of antimodal curves.

4.7. The dependence of the weather on its previous state. Lamarck (1800 – 1811, No. 5, pp. 5 and 8; No. 11, p. 143) knew the tendency of the weather to preserve its state. Even earlier Dalton (1793/1834, pp. 180 – 181) attempted to study the influence of the northern lights on the weather in Kendall (England). He compared the numbers of series of fine weather occurring after that event with those happening irrespective of the lights. The first number essentially exceeded the second and Dalton decided that northern lights were beneficial for the weather. He had not however taken into account the actually occurring series of fine weather.

Quetelet (1852 and elsewhere) and Köppen (1872) studied the tendency of the weather to persist by the elements of the theory of runs. So the application of that theory in natural science perhaps occurred first of all in meteorology.

4.8. Lamarck. He is meritorious for concrete results. No wonder! He studied meteorology during almost all his scientific life. Muncke (1837) had not realized Lamarck's findings, but Shaw & Austin

(1926/1942, p. 130) mentioned his *pioneer work in the study of weather*.

Lamarck noticed the indifference of both the public and the learned societies to meteorology and stated that, consequently (1800 – 1811, No. 4, p. 11), it became the domain of charlatans, that (No. 6, p. 5) it *did not yet exist*. He himself attempted to discover the laws governing the variations in the atmosphere whereas Humboldt preferred to study climate by the only possible at the time statistical approach.

Lamarck introduced and regularly used the term *météorologie-statistique* which was also the title of his article (1802) or *atmospheric statistics* (1800 – 1811, No. 11, p. 121). He (1800 – 1811, No. 4, pp. 153 – 159) listed the aims of this new direction of meteorology: the study of the climate and the atmosphere of a country although only as the first part of the statistics of that country (note here the connection with university statistics!) and only for the benefit of its political economy.

His contribution (1800 – 1811) consisted of eleven yearly publications with hardly successful predictions of the weather in France and general theoretical considerations.

5. Astronomy

5.1. The Solar system. The statistical approach to the structure of our system had already appeared in the work of Kepler and Newton and the Laplacean theory of probability was to a large extent connected to astronomy. He discovered or corroborated astronomical facts by statistically studying observations made by others, then justifying them, but as a rule only publishing the results obtained, see for example Laplace (1812/1886, p. 361).

Newcomb (1869) compared the theoretical (calculated according to the uniform distribution) and actual parameters of the orbits of minor planets, but was certainly unable to evaluate numerically his results. For him, those planets constituted a single statistical population {3}. Note that since 2006 asteroids and minor planets are called dwarf planets.

Poincaré (1896/1912, pp. 163 – 168) treated these planets the same way, as a population. He attempted to evaluate the total number of the still unknown asteroids but his deliberations were unworthy.

Much later Newcomb (1900) studied the motions of the asteroids. Both scholars made no difference between the probable and the mean values [of a random variable].

Drawing on his observations of 1826 – 1843, Schwabe (1844) established that the number of sunspots changes periodically and tentatively evaluated the period of the change, $T \approx 10$ years {3}. It was Humboldt's description of the work of Schwabe that turned the attention of astronomers to sunspots (Clerke 1893, p. 156).

Wolf (1859) collected the observations of sunspots from the mid-18th century and calculated instead $T = 11.1$ years. Then, applying observations made during 120 years he (Faye 1882) compared possible periods 9 years 6 months (2 months) 12 years 6 months. After analysing his results he concluded that there existed two periods, T_1 = 10 and T_2 = 11.3 years (least common multiple, 170 years). At present, strict periodicity is denied (as Schwabe prudently thought) with $T \approx 11$ years.

In 1776, Horrebow first suspected the periodicity of the sunspots (Wolf 1877, p. 654). Littrow (1836, p. 851) hesitatingly mentioned periodicity. Galileo is known to have discovered sunspots and to separate successfully their regular rotation with the Sun itself from their proper movement relative to the Sun's disk.

Daxecker (2004) studied the merits of Christopher Scheiner who had observed sunspots not later than in 1626 but their existence was known at least some centuries earlier. Humboldt (1845 – 1862, English translation of the pertinent volume, 1858, p. 64n) thought that *on the coast of Peru, during the garua* (whatever this means), sunspots could have been *highly probably* seen, *but no traveller has yet afforded any evidence of such appearances having attracted attention* […].

Nevertheless, here is Marco Polo (ca. 1254 – 1324), see Jennings (1985, p. 648): he described in passing his conversation with the astronomer Jamal-ud-Din, a Persian, and his team of Chinese astronomers. They discovered the sunspots (and apparently observed them repeatedly) when the *desert dust veiled the Sun*. This conversation took place in the last quarter of the 13th century somewhere near the present-day Chinese city Tianjin.

The influence of the sunspots (or generally of solar activity) on Earth magnetism was understood early enough, see Sabine (1852) and Faye (1873) who later (1878) somehow kept silent about that. In 1872 – 1880 other astronomers had studied the influence of the sunspots on cyclones and air pressure {5}.

5.2. The Michell problem. He (1767) attempted to determine the probability of a near vicinity of two stars provided that the stars were randomly (uniformly) scattered over the celestial sphere. His calculations were wrong, but his problem became classical and, interesting enough, he had applied geometric probability which only achieved universal recognition after Buffon (1777).

Many astronomers provided their own solutions of this problem, but the most interesting was the question of whether an event was produced by design or chance. Forbes (1849) noted that the existence of uniform randomness was better *consistent with a total absence of law* than the presence *of spaces of comparative condensation and regions of great paucity of stars* and that an introduction of any prior distribution was doubtful. In 1850 (p. 420) he asked himself which distribution can be called random but was unable to answer it. Then he experimented by throwing grains of rice on the squares of a chessboard and decided that its result confirmed his previous opinion. He could have referred to the celebrated Buffon investigation of the Petersburg game.

Newcomb (1859 – 1861, 1860, pp. 132 – 138) calculated the probability that some surface with a diameter of 1° contains *s* stars out of *N* scattered at random over the celestial sphere. He used the Poisson distribution. Then he (1860b, p. 438) decided that *some grouping* ought to be expected *as a result of a random distribution*. Newomb (1860b) repeated some of his arguments.

Much later Newcomb (1904, p. 13) indicated that *a chance distribution will more or less differ* from uniformity. And Boole (1851/1952, p. 256) called a distribution random *if it would appear to us that a star can be in any spot of the sky as likely as in another* (note the subjective approach). Any other distribution was *indicative*. These considerations had actually concerned the difference between an actual and theoretical distribution.

The history of the subjective probability and its applications deserves special studies; my attempt is Sheynin (2002).

Struve (1827, pp. xxxvii – xxxix) determined the probability that two or three stars are situated near to each other. A related problem concerned the distance between two random points on a sphere. Daniel Bernoulli in 1735 and Laplace (1812/1886, p. 261), then Cournot (1843, § 148) and Newcomb (1861b) solved it, but each of them interpreted that problem in his own way. Bertrand (1888, pp. 170 – 171) noted that likely/unlikely relative situation of two stars can be considered in different ways (not only a small distance between them) and concluded (pp. 4 – 7) that it was impossible to solve the Michell problem.

5.3. William Herschel: the starry heaven {6, 3}. About 1784 he began to count the number of stars seen through his telescope in different parts of the Milky Way. He assumed that the stars are distributed uniformly and that his telescope reaches the boundaries of the Milky Way. He intended to determine the relative distances to those boundaries, i. e., to study the form of the Milky Way. Distances, as he assumed, were proportional to the cube root of the pertinent number of the counted stars. Herschel only applied elements of sampling in one of its sections where a *glorious multitude* of stars prevented him to count them. Later Herschel abandoned his assumption and introduced a model of the distribution of the stellar distances. He (1817, p. 577) fixed the distances of the stars of each magnitude by concentric circles which allowed them to be anywhere within their shells that thus appeared, – allowed randomness alongside determination. That stellar distances have little or none at all connection with distances was then not known. Kapteyn (1906, p. 310) perhaps was the first to conclude that no real meaning can be attached to the motion of the mean distance of stars of a given magnitude. Herschel calculated the differences between the actual number of stars of each of the seven first magnitudes and their number which was proportional to the volume of the pertinent shell. The sum of these differences for the first four magnitudes was small and Herschel decided that his model was approximating that population well enough. However, the individual differences were too large and his model was hardly successful. His criterion resembled the main condition of the Boscovich method of adjusting redundant systems of linear equations.

While determining the velocity of the motion of the Sun, Herschel (1806) had to choose between the arithmetic mean and the median of a series of observations. On p. 358, perhaps following the then choice of Laplace, he opted for the median.

Herschel (1817, p. 579) reasoned about the size of the stars:

It may be presumed that any star, promiscuously chosen out of such a number [out of 14 thousand stars of the fist seven magnitudes] *is not likely to differ much from a certain mean size of them all*.

The sizes of stars are so enormously different (the stars belong to several different spectral types and do not constitute a single population) that their mean size is senseless. And *Ex nihilo nihil fit*!

Herschel is greatly meritorious for initiating stellar statistics. And I have no yet mentioned that he discovered a few hundred double stars and about two thousand five hundred nebulae and star clusters.

5.4. F. G. W. Struve. Drawing on statistical data, Struve (1847, pp. 83 – 93) attempted to prove that interstellar space absorbed light. His proof was not convincing and Newcomb (1861a, p. 377) rejected it as based on essential assumptions. However, the existence of that phenomenon was after all ascertained.

Struve (1847, Note 72) acknowledged that his assumption of a uniform spatial distribution of stars was inexact. But issuing from it, he calculated the maximal relative distances of stars of given magnitudes (see § 5.3 about the meaningless of that notion). Suppose that a certain portion of the sky contains *a* stars of the first five magnitudes and *b* stars of the first six magnitudes. Then the shell of the fifth-magnitude stars will have radius proportional to the cubic root of *a/b*. Like Herschel, he thus left room for randomness.

5.5. The proper motions of the stars {3}. In the 1830s – 1840s the study of the proper motions of hundreds of stars (although yet only in the plane perpendicular to the line of sight) had begun. Argelander (1837, p. 581) considered 560 stars with perceptible proper motions and determined the Sun's motion more reliably than it was achieved previously. Otto Struve (1842; 1844), then F. G. W. Struve (1852, pp. clxxxii – clxxxv) made the next steps. For the first of his studies the Royal Astronomical Society awarded O. Struve its gold medal (Airy 1842).

When studying the Sun's motion, astronomers beginning with Herschel thought that the peculiar motions of the stars (their motions relative to the Sun) were [random variables] and Kapteyn (1906, p. 400) called the random distribution of the direction of the peculiar motions *a fundamental hypothesis*.

Newcomb (1902, p. 166) assumed that the projections of stellar motions on an arbitrary axis are distributed proportionally to the normal law (he used this new term) as are their projections on an arbitrary plane whereas the motions themselves have distribution proportional to the law

$$f(x) = x^2 \exp(-x^2/c^2).$$

The first two laws were connected with the chi-squared distributions.

5.6. Statistical stellar astronomy. A large number of astronomical catalogues and yearbooks had appeared in the 19th century. Their preparation as also the compilation of star charts can be attributed to tabular statistics {6}. This statistical direction was even contrasted with theoretical constructions. Thus, Proctor (1872) compiled charts of the stars of the first six magnitudes showing their proper motion. Earlier he (1869) claimed the discovery of two stellar streams. I am unaware of whether Proctor's discovery was confirmed. It seems that he plotted 324 thousand stars and, anyway, he said so later (1873) and proudly stated (pp. 545 and 547) that he had not introduced any theories about the structure of the sidereal system.

A new feature of stellar astronomy appeared in the second half of the 19th century. Thus, Clerke (1890, p. 9):

Statistics are wanted of the distances and movement of thousands, nay millions of stars.

She referred to Hill & Elkin (1884, p. 191):

The great Cosmical questions to be answered are not so much what is the precise parallax of this or that particular star, but – What are the average parallaxes of those of the first, second, third and fourth magnitudes respectively, compared with those of lesser magnitude? [And] *what connection does there subsist between the parallax of a star and the amount and direction of its proper motion or can it be proved that there is no such connection or relation*?

Kapteyn (1904/1906, pp. 418, 419) reported that a study of the proper motions of the stars led him to believe in the existence of two star streams. Proctor (see above) stated the same, but I do not know whether Kapteyn thought of the same streams. At present, many streams (or perhaps streamlets) are known.

Kapteyn (1904/1906, p. 397) strove to describe statistically the sidereal system as a whole. He described the starry heaven by laws of distribution of parallaxes and peculiar motions of the stars and thus introduced random variables.

Newcomb (1902/1906, p. 302) remarked that the statistics of the stars

Commenced with Herschel […]. *The subject was first opened out* […] *through a paper presented by Kapteyn to the Amsterdam Academy of Sciences in 1893*.

He defined stellar statistics as a science of the unity of structure throughout the whole domain of the stars. Owing to him (Newcomb, p. 303), *we are able to describe the universe as a single object*.

Kapteyn initiated an international plan for a study of the stellar universe by sampling. He (1906, p. 67) quoted a letter of 1904 from an eminent astronomer, Edward Pickering whose recommendation to apply what amounted to stratified sampling he had been carrying out.

I do not describe the work of Seeliger (Paul 1993).

Newcomb highly appreciated Pearson. In a letter to him of 1903 he (Newcomb. Univ. College London, 773/7) wrote:

You are the one living author whose production I nearly always read when I have time and get at them, and with whom I hold imaginary interviews while I am reading.

In the beginning of the 20th century (1908 – 1910) Pearson had attempted to introduce correlation analysis into astronomy and published six relevant papers, but he was not sufficiently acquainted with the literature and barely interested astronomers. During the same period Julia Bell published two papers on the same subject in the *Monthly Notices of the Royal Astronomical Society*. She also was co-author of one of Pearson's papers.

6. Physics

6.1. Introduction. In 1738 Daniel Bernoulli qualitatively justified the elements of the kinetic theory by stochastic considerations. In essence, this discipline, as understood at present, originated in mid-19th century when the statistical method penetrated physics {4}. Khinchin (1943/1949, p. 2) maintained that Maxwell and Boltzmann applied

Fairly vague and somewhat timid probabilistic arguments that *do not pretend here to be the fundamental basis, and play approximately the same role as purely mechanical considerations.* […] *Far reaching hypotheses are made concerning the structure and the laws of interaction between the particles* […]. *The notions of the theory of probability do not appear in a precise form and are not free from a certain amount of confusion which often discredits the mathematical arguments by making them either devoid of any content or even definitely incorrect. The limit theorems* […] *do not find any applications* […]. *The mathematical level of all these investigations is quite low, and the most important mathematical problems which are encountered in this new domain of application do not yet appear in a precise form*.

His statement seems too harsh, written from the standpoint of statistical mechanics of the mid-20th century. Then, I believe that it was partly occasioned by Boltzmann's verbose style of writing. Third, physicists certainly applied the law of large numbers indirectly. Fourth, Khinchin said nothing about positive results achieved in physics (formulation of the ergodic hypothesis, use of infinite general populations, Maxwell's indirect reasoning about randomness). My first remark is indeed essential; here is an extract from Maxwell's letter of 1873 (Knott 1911, p. 114):

By the study of Boltzmann I have been unable to understand him. He could not understand me on account of my shortness, and his length was and is an equal stumbling block to me.

And Boltzmann (1868/1909, p. 49) owned that it was difficult to understand Maxwell's *Deduktion because of its extreme brevity*.

Statistical mechanics could not have appeared unless and until the kinetic theory with its mathematical shortcomings had been established. See Truesdell (1975) and Brush (1976).

6.2. Clausius. He (1857/1867, p. 238) stated that the velocities of the molecules essentially differed from one another. True, even Boscovich (1758, § 481) stated something similar, but perhaps presumed that the differences were not large.

Then Clausius (1862/1887, p. 320) maintained that those differences were random. He (1858/1867) actually determined the distribution function $F(s)$ of that free path ξ and determined $\mathrm{E}\xi$. Moreover, $F(s)$ was infinitely divisible. A distribution function indirectly appeared in the correspondence of Huygens in 1669 (Sheynin 2017, § 2.2.2), then such functions were introduced by Poisson and applied by Davidov and Liapunov (Ibidem, § 8.2). They could have been easily derived in insurance by De Moivre (Ibidem, § 4.2-3). They began to be regularly used in the 20th century.

Clausius (1889 – 1891) also applied stochastic considerations to solve concrete problems, but it seems that his main merit was that he influenced Maxwell (1875/1890, p. 427):

Clausius opened up a new field […] *by showing how to deal mathematically with moving systems of innumerable molecules*.

6.3. Maxwell (1860) established his celebrated [normal] distribution of the velocities of monatomic molecules. He tacitly assumed that the components of the velocity were independent; later this restriction was weakened (Kac 1939; Linnik 1952).

I am now largely reprinting my text on Maxwell and Boltzmann (Sheynin 2017, § 10.8.5). I have only printed its few copies and it is also on my website (**S, G,** 10).

Maxwell left interesting statements about the statistical method in general, and here is one of them (1873b/1890, p. 374), cf. the statement of Liebermeister in § 2.2:

We meet with a new kind of regularity, the regularity of averages, which we can depend upon quite sufficiently for all practical *purposes, but which can make no claim to that character of absolute precision which belongs to the laws of abstract dynamics*.

The drafts of the source just mentioned (Maxwell 1990 – 2002, 1995, pp. 922 – 933) include a previously unpublished and very interesting statement (p. 930): abandoning the *strict dynamical method* and adopting instead the statistical method *is a step the philosophical importance of which cannot be overestimated*.

Maxwell gave indirect thought to randomness. Here is his first pronouncement (Maxwell 1859/1890, vol. 1, pp. 295 – 296) which was contained in his manuscript of 1856 (1990 – 2002, 1990, p. 445), and it certainly describes his opinion about that phenomenon:

There is a very general and very important problem in Dynamics […]. *It is this – Having found a particular solution of the equations of motion of any material system, to determine whether a slight disturbance of the motion indicated by the solution would cause a small periodic variation, or a derangement of the motion* […].

Maxwell (1873a, p. 13) later noted that in some cases *a small initial variation may produce a very great change* […]. Elsewhere he (report read 1873, see Campbell & Garnett 1882/1969, p. 440), explained that in such instances the condition of the system was unstable and prediction of future events becomes impossible. Maxwell (Ibidem, p. 442) provided an example of instability of a ray within a biaxial crystal and prophetically stated (p. 444) that in future physicists will study *singularities and instabilities*.

In a manuscript of the same year, 1873 (Ibidem, p. 360), Maxwell remarked that

The form and dimensions of the orbits of the planets […] *are not determined by any law of nature, but depend upon a particular collocation of matter. The same is the case with respect to the size of the earth*.

This was an example illustrating Poincaré's statement concerning randomness and necessity, but I ought to add that it was not sufficiently specific; the eccentricities of planetary orbits depend on the velocities of the planets. See Sheynin (2017, § 7.3) where I noted an unbelievable mistake made by Laplace in this connection.

And here is Maxwell's position (1875/1890, p. 436) concerning randomness in the atomic world:

The peculiarity of the motion of heat is that it is perfectly irregular; […] *the direction and magnitude of the velocity of a molecule at a given time cannot be expressed as depending on the present position of the molecule and the time*.

6.4. Boltzmann. At the very end of his life Maxwell (1879/1890, pp. 715 and 721) introduced a definition for the probability of a certain state of a system of material particles; in later terminology, he preferred the time average probability whereas Boltzmann (1868, § 3) preferred the time average probability. Like Maxwell, Boltzmann (1887/1909, p. 264; 1895 – 1899, 1899, Bd. 2, p. 144) used the concepts of fictitious physical systems and infinite general population.

Boltzmann also maintained that both definitions were equivalent. I do not dwell on the later considerations about the ergodic hypothesis. Zermelo (1900, p. 318) and then Langevin (1913/1914, p. 3) independently stressed the demand to provide a definition *correcte et claire de la probabilité* (Langevin).

Boltzmann (1896/1909, p. 570) stated that the [normal law] followed from equal probabilities of positive and negative elementary errors of the same absolute value. His was of course an unworthy formulation of the central limit theorem although he respected the theory of probability. Thus (1872/1909, p. 317)

An incompletely proved theorem whose correctness is questionable should not be confused with completely proved propositions of the theory of probability. Like the results of any other calculus, the latter show necessary inferences made from some premises.

And again (1895/1909, p. 540): the theory of probability

Is as exact as any other mathematical theory if, however, *the concept of equal probabilities, which cannot be determined from the other fundamental notions, is assumed*.

From 1871 onward Boltzmann had been connecting the proof of the second law of thermodynamics with stochastic considerations. Thus, he (1872/1909, pp. 316 – 317) declared that the problems of the mechanical theory of heat are also problems of probability theory. Then, however, he (1886/1905, p. 28) indicated that the 19th century will be the age of *mechanical perception of nature, the age of Darwin*, and (1904a/1905, p. 368) that the theory of evolution was understandable in mechanical terms, that (1904b, p. 136) it will perhaps become possible to describe electricity and heat mechanically.

A possible reason for his viewpoint was that he did not recognize objective randomness. Another reason valid for any scholar was of course the wish to keep to *abstract dynamics*, see the opinion of Hertz (1894, Vorwort):

Physicists are unanimous in that the aim of physics is to reduce the phenomena of nature to the simple laws of mechanics.

I add yet another indirect reason. Although Boltzmann, in his popular writings, referred to many scientists, he never mentioned Laplace (1814).

And here is a lucid description of this point as far as Boltzmann was considered (Rubanovsky 1934, p. 6): in his works

Randomness […] *struggles with mechanics. Mechanical philosophy is still able* […] *to overcome randomness and wins a Pyrrhic victory over it but recedes undergoing a complete ideological retreat.*

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Abbreviation: Arch. Hist. Ex. Sci = AHES

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IV

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The theory of statistics in its present state with a short history of statistics

Translated by Oscar Sheynin

To scorn theory means to claim the right to act exceedingly ignorantly without knowing what will happen and to speak without understanding what is said
Benjamin Constant[1]

Berlin
2017

Теория статистики

в настоящем состоянии
с присовокуплением

Краткой истории статистики

Сочинение Александра Ободовского

ординарного профессора статистики
при Главном педагогическом институте

Mépriser la théorie, c'est avoir la prétention
excessivement orgueilleuse d'agir sans savoir ce
qu'on fait et de parler sans savoir ce qu'on dit
Benjamin Constant

Санктпетербург
В типографии Конрада Вингебера

1839

Contents

General Introduction

In this century, tireless investigations in the field of political sciences had beneficially influenced statistics. In spite of Lüder's scurvy tricks and threats[2] these sciences, based [only?] on the requirements of the human spirit, could not have perished. The truth triumphed and the Achenwallian – Schlözerian idea came even nearer to light[3] and statistics once more took that honourable place among the political sciences on which it was put by the immortal Schlözer.

Everyone finally became convinced in that political measures cannot be appropriate if not based on statistical data. However, for its triumph statistics is only obliged to its theory. No one anymore doubts that in higher educational institutions the theory ought to constitute the main and essential part of a course on statistics since it alone provides a proper view of this science and directs it to thorough and systematic investigations.

Indeed, only the theory invests independence to statistics and discovers invariable elements in this science. For statistics, theory is like a soul is for the body. Material statistics is similar to an unmeasurable and incessantly billowing ocean and all that, which is studied about it in universities, would have only been a fruitless coastwise navigation. Indeed, the instructor ought to teach his listeners statistics itself rather than exercise them in the difficult art of discerning, valuing, collecting and arranging statistical data. It is that goal to which the theory guides the beginners.

The aim of my book is to represent the theory of statistics to the beginners in its present state and, at the same time, to acquaint them with the historical destiny of statistics[4]. The publication of the theory seemed to be all the more necessary since not a single contribution in our national literature had appeared after 1809[5] in which that science was treated systematically as required at present.

Notes

1. Henri-Benjamin Constant de Rebeque (1767 – 1830) was a writer on politics and religion. O. S.

2. Lüder (1817, p. v) formulated his aim as destroying statistics and politics which is closely connected with it and likened statistics with astrology (p. ix). I am unaware of Lüder's influence on the development of this science. Anyway, Schlözer had not mentioned that contribution; Obodovsky did (see below) but dismissed it. O. S.

3. Obodovsky had not explained what exactly did the *immortal Schlözer* achieve in statistics. I consider his main statistical work (1804) barely useful, see its translation on my website www.sheynin.de downloadable file 86, Introduction. O. S.

4. Obodovsky had not shown the *historical destiny* of statistics. O. S.

5. In 1809 Hermann (Herrmann) published (in Russian) his *General theory of statistics.* See a discussion of its first chapter in Sheynin (2014/2016, pp. 9 – 10). O. S.

The Theory of Statistics

Introduction

1. On science in general

A mind, after preparing itself for highest activity, shuns scattered and fragmentary notions. Seeking everywhere unification, it searches according to natural tendency for such an elevated point from which it will be able without distraction to cast a glance into the depth of the mastered knowledge. Being fully cognisant of its triumph and power, it assimilates and surveys that knowledge. To attain this aim, the mind collects homogeneous notions under particular ideas into general notions and subdues many similar truths under one single main idea.

Thus each science is created. But how many notions should constitute a science? This is determined by the powerful human spirit and some superior gift of construction peculiar to a creative genius.

2. On theory in general and on the theory of statistics

Sciences are subdivided into philosophical and experimental or historical depending on their belonging either to [studies of] mental or material objects. Any philosophical science is called a theory if expounded without any applications. In each experimental science subject and form are necessarily discerned. Its subjects are facts or data (§ 22) whereas the form is the method of uniting those facts or data.

And so, each experimental science has two parts, material and mental[1] and together they constitute the system of the science. The latter part is its theory and in this sense a theory only belongs to the system of a science. But the theory is also understood as an investigation of the properties and components of a science in general and its peculiar features. In statistics, both these studies are adopted jointly.

Notes

1. This (which is also mentioned below) contradicts the above. O. S.

3. The necessity of a theory and especially of a theory of statistics

The critical spirit of our century proves that a theory is needed for each experimental science. Indeed, each requires it since only a theory can completely separate a science from other sciences[1]. Only it provides independence and internal order, indicates its merits, goals and usage and teaches us how to cultivate it.

Experience convinces us that a proper theory greatly influences the success of the science and its practical value. Statistics especially needs a robust and thorough theory because of the peculiar property of its subject which is fused together from material and mental components. Being so complicated, its subject often became an occasion for misunderstandings and delusions which are so pernicious for each science.

Note

1. Sciences are interconnected. Mathematics, for example, is connected with physics, biology and economics, to mention only three other sciences. And, for an example, William Herschel originated stellar statistics. O. S.

4. Subdivision of the theory of statistics

We understand the theory of statistics as the investigation of the properties and accessories of statistics understood as a science in general and also of its peculiar features and its system. Its properties, accessories and peculiar features are perceived by its subject or the problems which it solves, by its name, definition, boundaries and usefulness.

The system is determined by that definition. Cultivation of a science requires the knowledge of the methods of acquiring and expounding statistical information.

Part 1. The Notion of Statistics

5. The aim of statistics in general

A state is a society which aims at security[1] and welfare[2], both physical and mental. It is a moral organism[3], i. e., a system of moral powers directed towards the aim required by reason.

Notes

1. Those people who are united into societies because of a natural inclination, cannot enjoy unbounded freedom of action. Indeed, such freedom will become a source of mutual resentment and oppression. Each member of a society certainly ought to restrain his freedom of action so that he will not prevent the actions of other members. This means that a member of a society only has a *right* to act without restraining others. However, in such a society with each being his own judge most governing will be the right of the strong. [Cf. Schlözer (1804, §§ 11 and 12). O. S.]

Moreover, people will be unable to agree about the boundaries of the freedom of action since they differ in intellectual abilities, moral qualities, temperament etc. The *law of rights* is therefore needed which should certainly physically prevail over their denial and eliminate all hindrances to the security of *rights*. Only such societies may be called states which have *the law of rights* and consequently *security*. A. O.

In the beginning of § 39 and in § 51 Obodovsky called the state a political body which is not a moral organism at all! O. S.

2. The aim of a state does not only consist of security for each. When entering a state, a man brings along not only all his abilities and forces, but the general final aim of the entire human existence, morality and welfare as well. The aim of a state should therefore also to the same extent include decrees necessary for the moral perfection and physical comfort, or, in one word, decrees, directed towards the welfare of the citizens. At the very least, the aim of a state ought not to oppose the aim of humanity.

The law of rights or security issues from the government whereas welfare is rather the concern of the citizens. An enlightened government helps the citizen to attain welfare only in such cases in which their private forces are unable to overcome the encountered difficulties. A. O.

Concerning the aim of the human existence see Note 1 to § 41. O. S.

3. Just like an individual, a state, considered as a whole, consists of body and soul; it is a moral person or a complicated man. Its combined members are its body, the love of the Tsar and Fatherland is the soul, and the heart is the Church. The spirit of government or the totality of the actions of all its moral forces is revealed in its political life.

The aim of human existence is the highest possible harmonious development of the forces granted humans by God [This contradicts the statement in Note 2. O. S.]. The aim of the state concerning its composition is the most perfect security as the first condition of political existence, just like health of the human body is the condition for perfecting the soul. The aim of an unbroken political life of a state is welfare, and since the state consists of reasonable beings, its welfare needs material comfort, the people’s wealth and, just like a moral organism, it also needs art, science, morality, religion. A. O.

In the beginning of § 39 and in § 51 Obodovsky called the state a political organism, see also § 7, but in § 27 he mentioned moral organism once more. O. S.

6. Continued

In the physical world, everything is interconnected like cause and effect according to the law of necessity, and the aim of a physical organism is attained by the same law. In the moral world, the connection between cause and effect is established by reason and the aim is achieved by freely chosen methods. Those methods infinitely differ not only as such but also in the extent of their effectiveness and therefore lead to differing results.

However, if the methods are chosen prudently rather than at random, the aim of the pertinent investigation will be necessarily and most clearly seen[1]. But even then there remain so many and so different methods that the attainment of the aim will not be equally successful. Under given circumstances the mind will consider as sufficient only definite methods which still can infinitely differ[2].

Notes

1. The grammatical construction of that phrase was faulty and the translation is only probable. O. S.

2. All this reasoning seems artificial and meaningless. O. S

7. Continued

What was said in § 6 about the moral organism can be easily adapted to a political organism. When considering some particular state we should first of all imagine its aim whereas the methods for attaining that aim are known to it. The study of these different methods and their actual application as well as the results obtained provides a very fruitful subject for reflection.

If, in the course of such investigations, we begin to discover general notions in our [acquired] arsenal of knowledge and arrange them according to their interconnections or systematically, we will thus create a new science. Such a science really exists and is called *statistics*[1].

Note

1. This reasoning seems too simplified. O. S.

8. An exposition of the name *statistics*

The origin of the word *statistics* as a designation of a science whose aim we have determined is obscure. It was probably borrowed from the word *statista* first applied by Oldenburger, a professor at Geneva [at that time] in 1675 in his Itinerario Germanica Politico[1] for denoting the merits of V. L. Seckendorf. That latter (1756, Introduction) stated [translation from the Russian translation]:

I had no intention of depicting a general German politics[2] *or rules of governing a state. My aim and intention were, to describe the condition of most German states in their proper and well organised way. But I was the first to venture such an enterprise so that my bold action or my defects will prompt others to do something better*.

Such an explanation clearly shows that Seckendorf thought that his work absolutely differed from politics. Had Oldenburger called him a politician, he would have been censured for looking at him from an inappropriate angle. And since Seckendorf specifically based himself on the word *status*, Oldenburger who intended to define Seckendorf's moral quality (?) should have invented a new word, and called him an *egrerius* [honourable] *statista Christianus*[3].

Notes

1. Germany had not existed yet but in those times that word denoted the German world in general. O. S.

2. See Oldenburger (1675, t. 4, p. 824) and Klotz (1821). A. O.

3. Schlözer (1804, p. 3); Klotz (1821, p. 11ff); Holzgethan (1829, p. 1). Hassel, Gassel (1822a, p. 1) supposes that the word *statistics* was compiled from the Latin *status* and the Greek *aritmetika*. Some authors advise others to write *statistics* with a double *a* since they believe that that word originated from the German *Staat*. A. O.

Lovric who wrote § 1 of my essay (2011) discovered that the word *statistics* or similar words had appeared several times before 1600 although perhaps not to denote a science. But neither did it apply to a science for Oldenburger! And Obodovsky himself (beginning of § 9) noted that much time had to pass until this happened. O. S.

9. Continued

The word *statistics* had thus been composed about the mid-17th century but rather much time had to pass before its derivative, *statistics*, became used for designating a science[1]. Achenwall, in the mid-18th century, was the first to apply the word *statista* as a noun designating a science. Although it did not appear in the title of his book nor was he its inventor, as everyone believes [contrary to what everyone … ?], we should regard it as a merit that he introduced it into general usage. Schlözer [1804, § 1] called it barbarian and corrupted (*vox hybrida*)[2] but in the newest languages there is no other word precisely answering the required notion. And so, the previous allegoric Latin *notitia rerum publicarum, notitia orbis imperantis, notitia statuum* remained in use also by those authors who still write in Latin. And that word, *statistics*, was introduced into all European languages[3].

Notes

1. Schlözer [1804, § 1] testifies that it was first used as an adjective by Thurmann (1701), than in Schmeizel's (Шмейцель) 1725 announcement of his lectures which he called *Collegium Statisticum*. However, Thurmann's *Bibliotheca Statistica* is known to be *Bibliotheca Politica* [as previously noted by Schlözer – O. S.]. And, having only an announcement, we are unable to say whether Schmeizel meant statistical lectures in our sense. A. O.

2. Humboldt (1815, p. viii) wrote *political arithmetic or, in latino barbare, statistics*. And he wrongly equated both sciences. O. S.

3. In France, it first appeared in a book of Brion de la Tour in 1709 and, the same year, in England, in *Monthly Review*. In Russia, still earlier (Noveishee 1795). See *Vseobshchaia* (1809). A. O.

10. The need to define statistics

According to the meaning of the word, it is the study of the conditions of some state. However, this explanation is not sufficient for precisely understanding statistics as a science. For statistics to solve systematically its problem (§ 7), we ought to define it since only a definition provides an exact understanding of a science. Not only the independence of a science depends on its definition, but its internal order and its distinction from other sciences and, finally, the very viewpoint on the science from which authors had attempted to deal with it in different times.

A perfect notion or definition is, as Butte formulated it, a sanctuary in which there lives the main, the general idea which serves as an Ariadna's clue. From that clue as from an embryo harmoniously develops a definite notion of a complete statistics. And who denies the need of a definite notion of statistics, thus certainly deprives it of its worth among other sciences and leaves it without any systematic order.

11. What kind of a definition of statistics should there be?

A definition of statistics, just as of any other science, ought to conform precisely to its subject, represent neither more nor less. It should signify the content of that subject[1] and show the creative idea of the science in all its worth; should eliminate all the alien but collect the homogeneous; impart originality, completely separate it from other sciences[2] and indicate the path to the internal connections of those diverse matters[3] which ought to be united into a single harmonious whole.

A true [a proper; I will not repeat this remark – O. S.] definition of a science should also represent the notion of its entire structure in an extremely abstract way so as to include superior indications. Finally, that definition ought to represent the matter created by the idea and include that idea itself. Then, who grasps the definition in all its completeness will be himself able (certainly under favourable circumstances) to create a science[4]. It should represent the measured magnitude (?) and provide a scale for the measurement.

Notes

1. Indeed, ancient mathematicians defined a point as an object without dimensions, but modern science (logic and mathematics) require a positive definition and have to leave the point without any definition at all. O. S.

2. See Note to § 3. O. S.

3. Diverse but homogeneous! O. S.

4. The same science (statistics) anew? Anyway, this statement is certainly far-fetched. Concerning the next sentence see Note 1 to § 18. O. S.

12. Authors disagree about the definition

Thus, then, the definition of statistics should be for placing it among other sciences and enjoying originality. However, not every author, for example, Malchus (1826, p. 6), believe that such a definition is really needed, but they base this conclusion on stating that the subject of statistics is facts and their description, that statistician is only a reviewer. For them, a systematic development of the facts by issuing from superior elements or the submission of the former to the latter is really foreign to the notion of statistics.

Those who think so forget that the data which concern a state are not yet statistical; they become statistical when considered from a certain viewpoint. And this condition is only met by a definition which includes an idea of a science.

13. Continued

For about a hundred years now, the scientific world is regarding statistics as a science[1] but almost each statistical contribution includes a definition of statistics more or less deviating from those provided earlier[2]. Many of them are actually descriptions which had been satisfied with minor and accidental indications and represent a science [represent statistics] either too extensively or too narrowly.

Some statisticians (among the latest of them are Malchus [see § 12] and Schubert) have not offered any definition at all. Perhaps they agreed with those which had appeared previously, or understood that it was impossible to squeeze all that which they included in their contributions into the narrow confines of a definition.

Notes

1. At least after 1839 (after the publication of this book) several authors (Fox 1860, p. 331; Alph. DeCandolle 1873, p. 12; Miklashevsky 1901, p. 476) stated that statistics was only a method. O. S.

2. Lüder (1817, pp. 98 – 109) collected many definitions and sharply discussed the contradictions between them. A. O.

14. A survey of the most important definitions of statistics including those which exist nowadays

A criticism of all the existing notions of statistics would be fruitless and uninteresting. However, before approaching the true definition of statistics it is necessary to survey the definitions of the most important authors. We will then find out what was achieved in statistics as a science and become able to compare the present and the previous views about statistics.

Concerning definitions, the authors can be divided into five groups. (a) Those who think that statistics is the cognition of the real conditions and quality of states. (b) (The first class.) Others, and especially French authors, call statistics the study of the power and might of states. (The second class.) (c) Niemann and Malchus equate statistics with its theory. (d) (The third class.) According to their definitions, Sinclair and Gioja do not recognize statistics as a political science. Sinclair believes that statistics is a study of the conditions of some territory aimed at discovering the degree of the welfare of its inhabitants and the means for increasing it. (e) Gioja defines statistics of some territory as all the information which can be useful in general to anyone or to most inhabitants, or to the government. (The fourth class.) Finally, Achenwall and his followers call statistics the cognition of the remarkable features of the state. (f) (The fifth class)*.

Notes

*The subdivision into classes is not easy to understand. I have left the letters (a), (b), … as inserted by the author rather than replacing them by numbers. O. S.

(a) *Noveishee* (1795) admits two classes of definitions, see § 31. Gess (Гесс): in his *Comments* (p. 10) admits three classes; Butte (1808, p. 197), four and Holzhethan (1829, p. 14), six classes; Klotz (1821), eight classes. On the contrary, Malchus (1826, § 2) considers the subdivision of definitions into classes superfluous, their distinction illusory rather than essential since all the authors, as he thinks, have the same aim although approach it from different directions and admit similar distinctions in science. [The author repeats *science* many times; did he mean statistics? O. S.] But still, who can offer the best definition of a science?

(b) Here we have **Conring**: statistics is [I am not repeating these two words – O. S.] a complete cognizance of mostly the present and the previous conditions of a state and, as far as possible, its future condition. **Toze** (1762): the recent history of a state and a description of its present condition. **Lüder** (1792): it represents the conditions of a state at present or at a definite time. **Mader** (1793): it is knowledge of the real condition of a state. **Sprengler** (1793): the historical science which entirely and reliably describes the present or normal situation of a people. **Lucka** (1796): Practical statistics is the cognizance of the real quality of a state in all its parts. **Mone** (1824): the representation of the conditions of a state at the present and continuing time. **Koch-Sternfeld** (1826): cognition guided by theory and experience of the recent conditions of a state. That cognition is necessarily combined with the study of its organic and real basic power and its essential change due to events and political rules. **G. Boulgarin**: a science of the recent conditions of states represented by the entire display of their internal and external life.

(c) 1) **Peuchet** (1805): a science of the real forces and means of the power of some state. 2) **Mannert** (1805): a representation of the forces of some state. 3) **Donnant**: a science which considers the physical, moral and political forces of some territory.

4) **Fischer**: a science which teaches us how to study the forces of a state, to judge and describe them according to their properties, unification and usage. 5) The **Hassel (Gassel)** definition can be included here: a description of states according to their structure and internal and external actions. 6) **Zizius:** a systematic representation of the data from which we are able to study the conditions of the real political might of some state.

(d) Both authors call the material part of statistics *Statecraft* (Staatskunde) and its formal part, *statistics.* **Niemann** (1807, pp. 7 and 8):

Statecraft is neither a mass of numbers or information collected without any plan, nor a unification of that which seems remarkable according to the tastes of any individuals. It is *a correct representation of the state authorities and order in that state and of the civil way of life under their influence*.

The statecraft, thus understood, is a special subject for study. *The representation of a state has its own rules for both considering it from the single proper point and for its usage to attain the supreme aim*. Statistics is the totality of those rules.

(e) See **Sinclair** (1791 – 1799, vol. 20, p. XIII):

Many people were at first surprised at my using the new words <u>Statistics</u> and <u>statistical</u>, as it was supposed that some term in our own language might have expressed the same meaning. But in the course of a very extensive tour through the northern parts of Europe, which I happened to take in 1786, I found that in <u>Germany</u> they were engaged in a species of <u>political</u> inquiry to which they had given the name of <u>statistics</u>; and though I apply a different idea to that word, for in Germany statistical meant an inquiry for the purpose of ascertaining the <u>political strength</u> of a country, or questions respecting <u>matters of state</u>; whereas the idea I annex to the term, is an inquiry into the state of a country for the purpose of ascertaining the <u>quantum of happiness</u> enjoyed by its inhabitants, and the <u>means</u> of its future improvement; yet as I thought that a new word might attract more public attention, I resolved on adopting it, and I hope that it is now completely naturalised and incorporated with our language.

[Schlözer (1804, § 5) quoted that passage and noted that Sinclair *certainly had not read a single German statistical handbook*. He also explained practical politics: it is the *doctrine of governing the state or the science of governing*. O. S.]

(f) 1) **Achenwall** (1768, fifth edition [of his book of 1749]: the totality of the real remarkable features of some state [kingdom] or republic. In the broadest sense it is the structure (Staatsverfassung) of that state. And the science of that structure is statistics. 2) **Schlözer** [1804, § 14]:

Statistics of a land and people is the embodiment of the remarkable features of the state.

3) **Remer:** the science of the structure (Verfassung) of various states. 4) **Meusel:** the statistical representation of the quality and structure of a state. 5) **Goes** (1806): [Obodovsky repeats the title of that book]. 6) **Schnabel:** a statistical representation of the real situation for fostering the art of managing the state. 7) **Heim, Ziablovsky** (Гейм, Зябловский): a thorough cognizance of the real remarkable features.

Druzhinin (1963, p. 67) mentioned E. F. Ziablovsky (1763 – 1846), professor of history and geography, later, of statistics in Petersburg and called him a reactionary. O. S.

15. Criticism of the definitions of statistics[1]

When we consider these classes of definitions and recall what was said in § 11 about a perfect notion of science, we easily see that all of them are more or less unsatisfactory. The definitions of the first class provide statistics with a superfluous scope. Indeed, statistics will then include all the details of the description. On the contrary, the definitions of the second class are too narrow and one-sided since forces are only natural abilities and mean something positive whereas statistics considers negatives as well. In addition, statistics cannot avoid studies of the established order or management or enlightenment of the state whereas forces and might only have to do with its external relations..

Niemann and Malchus unjustifiably deny the adopted nomenclature and separate the theory of statistics from statistics itself. The theoretical part of statistics which we, together with Schlözer, call its theory, had not yet achieved a degree of perfection sufficient for separating it[2]; again, the two parts of statistics thus separated will be based on the same main idea and cannot therefore be different sciences. Finally, how then to name the science in which the practical and the theoretical part are fused? The definition offered by Sinclair shows that he had not thought about studies of states and his contribution only belongs to statistics by name.

Most satisfactory among all the definitions are those suggested by Achenwall and Schlözer and they therefore deserve to be specially studied.

Notes

1. See a most detailed criticism in Lüder (1817, p. 98) and Klotz (1821, p. 19). A. O.

2. A most extensive attempt of such a separation is Gioja (1838). A. O.

16. Continued

The definition of statistics as a cognition of the remarkable features of a state clearly shows that statistics has to do not with physical, geographical, literary, or technical remarkable features but with those of the state. However, a question remains: What is included into them? Or, which is the same, why can some information be called statistical? Achenwall himself was not satisfied by that expression and interpreted it (p. 5):

Infinitely many objects indeed exist in each state. Some of them noticeably influence its welfare, either furthering or hampering it. Such objects can be called remarkable features of the state.

Schlözer (1804, §§ 12 and 13) explained the situation more skilfully and in great detail: there are

Descriptions by the physicist, the geographer, the naturalist (botanist, zoologist, mineralogist[1]), *by the historian, antiquarian, economist, publicist, teacher of religion and by a dozen others, each keeping to his own field. Even in a tiniest state they will find sufficient material for description.* […]

For each realm and each of its provinces there can appear 20 or still more such conceivable special skilful descriptions. […]

All the data for which the statistician is searching, should also be in those 20 special descriptions if they are supposed to be complete. However, since each compiler had his own aim, I imagine one other aim which no one of the former compilers had but which is of a convincing importance and worth. The scientist who studies the state, either a practical worker or a theoretician, enters as the 21st man with the intention to elicit only those features which <u>apparently or conceivably influence the welfare of a state in a larger or smaller measure</u>. He takes for himself only these and orders them properly one after another.

Note

1. In 1857, the International Statistical Congress (Congrès 1858, pp. 390 – 397) published a questionnaire naively entitled *Eléments qui les scicnces naturelles fournir à la* [ought to provide] *statistique*. See also Sheynin (1980, p. 332). O. S.

17. Continued

Who reads Schlözer's (1804, §§ 14 and 15) explanation of remarkable features of the state certainly will not deny the truth [the propriety] of the Achenwall – Schlözer definition. However, after considering in all rigour the property of the definition, we will have to agree that their choice is not understandable without a special interpretation.

Both Achenwall and Schlözer believed that the statistical data are distinguished from non-statistical by their influence on the welfare of the state (§ 16) and both also agree that welfare is the aim of state. It follows that statistical data are only those which influence the aim of the state[1]. Therefore, the aim of the state is the truest and initial indication of a statistical datum. And since a definition ought to offer a notion as abstract as possible (§ 11), it also follows that they both only gave an understandable explanation or description of statistics rather than its definition. Indeed, their definition includes lower derivative information. Only a rigorous, true definition can be useful for cultivating science. The description offered by Achenwall and Schlözer is unsatisfactory although correct and true.

Note

1. The following definition is also relevant here:
Statistics is a science which considers the actual condition of a state to show the extent of its security and welfare at some definite time. A. O.

18. The true definition of statistics

Now it seems easy to express properly the Achenwall – Schlözer explanation of a remarkable feature of a state. Butte had fulfilled that important service. Authors who agreed with this definition apparently belong to a special class but actually they are Achewall's followers. Butte himself did not consider his definition new, he only indicated that he adapted the Schlözer definition to the requirements of science. He formulated that definition so that it represented science in the highest possible abstract way. Being a measurable magnitude it also offers a scale for the measurement[1]. The essential difference between the two definitions consists in that Butte, instead of a minor, included the highest indication, and along with it other indications concealed in the notion of remarkable feature of the state which occur in the definitions of other authors in a scattered way.

We acknowledge the Butte definition[2] in the following form:

Statistics is the systematic representation of those data which allow a thorough discovery to what extent had the state attained its aim at some definite moment understood as the present.

Notes

1. Measurement and scale are also mentioned at the end of § 11, but remain mysterious. O. S.

2. *Statistik ist die wissenschaftliche Darstellung derjenigen Daten, aus welchen das Wirkliche der Realisation des Staatszweckes gegebener Staaten in einem als Jetzzeit fixierten Momente, gründlich erkannt wird*.

19. Explanation of the true definition of statistics

However clear that definition is all by itself, an explanation ought to be attached to it to prevent misunderstanding. Judging by the importance of definitions such misunderstandings are often dangerous for the success of science. An explanation seems all the more necessary since some authors had not quite agreed with the Butte definition and corrupted it by useless additions or gaps[1].

Note

1. For example, Lichterstern Tl. 1, p. 6 [see Bibliography] provides a very long definition. Klotz (1821, § 14, p. 25) translated the Butte successful definition into Latin: […]. However the [obviously, his] subdivision of the aims of the state into internal and external aims is wrong. A. O.

20. Continued

Statistics is a systematic representation. In general, science is a systematic totality of truths. It ceases to be a science as soon as it has no system, i. e., no order determined by a single main idea which unites all those truths and links them into a single whole. Without such an idea science naturally becomes disordered, lacks any plan and connections. We will then be liable to the danger of losing our way in an unmeasurable sphere of knowledge and include into the science such subjects which do not belong to it. What concerns science in general can be applied to statistics.

21. Continued

Statistics is a representation. Some authors say that statistics is a description but it represents measurable data by numbers which cannot serve for description. And if statistics concerns moral matters, it does not restrict itself by a simple description but offers a picture as clearly as is necessary for its goals.

In any case, statistics attempts to represent the aim of the state clearly and lively. It should therefore be called a picture rather than a description. This consideration shows that statistics is a historical science[1].

Note

1. This statement is not explained. Furthermore, it contradicts the end of § 22. O. S.

22. Continued

Those data. All the existing can be thought as phenomena liable to cause and effect and therefore as something created, or, just as something existing in time and space without any connections to cause and effect[1]. In the first case we have a fact, in the second instance, a datum. It is impossible not to agree that each fact can be a datum and vice versa. However, since each indication offers its own viewpoint, it is better to call statistical objects *data*. Here is an example: the territory of Russia. Who shows how Russia acquired its great territory which constitutes 1/6 of all inhabited land, understands that territory as a fact. On the contrary, someone who reports about it as about something given, has no need to enter into historical studies, he just says that its area is four hundred thousand square miles. In essence, statistics only collects data whereas the objects of history are facts[1].

Note

1. First, it seems that randomness does not exist (see the beginning of this section)! Second, history is not restricted to chronology, as noted by Schlözer (1804, § 26). And about twenty years later his son, Christian von Schlözer (Sheynin 2014/2016, p. 18) maintained that only narrow-minded people restricted history to chronology and believed that it does not need general principles. But my main comment is that Obodovsky had here (and elsewhere) excluded the discovery of causes and effects from statistics as well. An important addition is needed. In many cases cause and effect are immediately seen in the statistical data, and only one question then remains: why not report such cases, if essential for the state, to the authorities at once, why wait indifferently? Schlözer (1804, § 14, Item 3) all but failed here. O. S.

2. This conclusion seems artificial. O. S.

23. Conclusion

Which allow a thorough discovery. Thoroughness is required of each science and constitutes a necessary condition for any system of truths. It means depth of cognition; generality of notions which is able to discover mistakes in particulars; invariability which is often alien to the material part of science.

Misunderstandings about the thoroughness of statistics had prevailed and many authors had attempted to be called thorough statisticians by offering infinitely long series of numbers, or, by trying to be clear and therefore flooding statistics with many notions belonging to other sciences. Thoroughness in statistics does not consist in the knowledge of numbers or in borrowed explanations, but in proper distinction, estimation and arrangement of statistical data. An author can be called a thorough statistician if he knows the theory of statistics perfectly well as also the material matter based on it.

Given the variability of statistical data thoroughness consists of grasping the invariable elements of statistics which may serve as rules (?) for an entire life. Then any statistical investigation will be surely successful.

24. Continued

To what extent had the state attained its aim. We see here that in the strict sense statistics is a political science since its subject is the state. However, the state is also the subject of other sciences. The science of the state (Staatslehre) shows the ideal condition of a state, politics sets forth rules for the state to attain its aim, whereas statistics shows to what extent the state has attained it.

Here was the clear difference between statistics and the science of the state and politics. It also follows that statistics is an experimental science since its subject is not the ideal, but the real state. It only depicts the really existing without bothering about what could or should be. Nevertheless, we ask readers not to forget that an experimental science is only possible when it is arranged according to a general idea rather than blindly following experience. In this latter case observations are accidentally carried out without any plan, are not connected necessarily, and statistics is not a science anymore. In an experimental science, experience, so to say, is required by reason to answer its questions. Only then a unity and a system are possible.

Note also that the aim of a state can be either necessary or empirical. The former is that essential indication without which we cannot imagine a state. It consists of security and welfare (§ 5). Apart from this general purpose any state considered along with other states aspires, just as any indivisible unity, to attain the aim of its existence. This aim is assigned by its natural or acquired abilities or appears according to special rules adopted by its government.

A state can aspire to extend its trade as Great Britain, or, as France under Napoleon, to conquer other countries. Such aims are called *empirical*. Statistics, a science and therefore a child of reason, when depicting a state, should pursue only one aim which is grasped by reason rather than being empirical and accidental[1].

The state should regard as necessary everything contained in itself and apply it for checking everything. The scope of statistics is restricted by that necessary aim[2]. Only when having it before our eyes we are able to detect deficiencies by comparison. Indeed, in general a deficiency is discovered by comparison with what ought to be. Only the recognition of this aim can lead to unity of the material statistics without which a systematic statistics cannot exist. Nevertheless, given that unity, the statistics of states will not be uniform. On the contrary, even with the common character of the general ideas they will manifest an infinite variety since the aim of a government is attained by infinitely many methods (§ 6).

Notes

1. Foreign trade is hardly accidental for any state. O. S.

2. The study of foreign trade is beyond the scope of statistics? And the previous sentence is hardly understandable. O. S.

25. Continued

At some definite moment understood as the present. All the authors of statistical contributions agree that the subject of statistics is the information about the present time. It does not therefore study the past or the future. The former belongs to history, the latter, to philosophers and poets[1]. However, after understanding that under the pen of the statistician the present is determined by the past[2], we may think that statistics never solves its problems, or, simply, that it is an impossible science. On the other hand, those same authors believe that the notion of statistics should not reject the possibility of compiling the statistics of Greece at the time of Pericles; or of Rome at the time of Augustus; of the kingdom of the Franks under Carl [Carolus Magnus, 742 or 748 – 814, or Charles Martel, 686 or 688 – 741], or the Russian Empire of Peter the Great. So how to reconcile these contrary views? There exists only one means:

For compiling the statistics of some state we should imagine some arbitrary time, place and remoteness [from our time?] *in its life; mentally separate this* [moment?] *from the past and future; and thus construct an imaginary present*.

Thus it is done in contributions and universities in present-day Great Britain and France. However, if a statistician incessantly receives information about those states [certainly not about Great Britain or France!] its [their] present condition will still be imaginary. In such a way we say in 1839 about 1838 as about the present time.

Let us apply this method of imagining the present, separating it from the past and future, for any arbitrary selected moment of time. We will then act in the spirit of our science. A mental separation of some moment from past and future is its complete separation from time since time is going by whereas a statistical moment does not recognize any movement. Therefore, if statistics is called a historical science, it only signifies its contrariness to philosophical science.

History and statistics do not relate to each other as the past and the present, otherwise statistics will be, as it is usually said (Schlözer [§ 23 bis[3], Item 6]): *History is statistics flowing and statistics is history standing still*[4]. The mental separation of a statistical moment from time provides that invariability and constancy which, as it appears according to its materials, are not at all foreign, but belong to it inalienably as to any other science.

Time does not diminish the worth of a good statistics and Niemann justly called such contributions as Middleton's (1750) biography of Cicero, Gibbon's (1776 – 1788, Chapters 1 – 3 and 6 [of which volume?]) *History* or Voltaire's description of France at the time of Louis XIV [1638 – 1715] fragments of statistics which will always be read with pleasure.

Statistics of Russia, Prussia, … only exist when that viewpoint on science is adopted. It will always remain true that already during compilation and printing contributions, they present the past. The moment in the life of a state selected by the statistician represents the continuing condition of the state during which it is not subjected to any serious change. But how long is that period? As one of our honourable authors of statistical contributions, the late Hermann (Herrmann) put it:

I maintain that a good statistics shows the condition of the people at least for twenty years. Everything (in the state A. O.) *remains for quite a long time as it was. Objects are necessarily moving, but they always rotate about the same axis and their relations to each other remain without change.*

These relations are so invariable that the condition of one object can be judged by the condition of the other one even when numbers change by a few hundreds or a few thousands. They certainly do not change by millions.

However, the space [the period of time] which corresponds to the statistical moment cannot be the same for all states (Mone 1824). After a state had attained a certain level of development, its successes slow down. The state remains on that level for a long time especially if its natural situation hampers industrial activity and participation in world trade. On the contrary, states naturally beneficially situated and having the possibility to participate in that trade promptly develop their forces and under favourable circumstances grow and change and require an often repetition of general statistical studies.

Notes

1. Nowadays attempts are incessantly made to foresee the economic and/or political future of states. O. S.

2. Apparently: after statisticians study the past. O. S.

3. Schlözer mistakenly numbered his sections: numbers 23 and 24 appeared for the second time after number 24. So 24 bis means the second number 24. O. S.

4. Schlözer (beginning of § 24 and § 26) also stated that statistics is a part of history. Again (§ 14, Item 3 and Note 4; §15, Item 12) it is necessary to compare one state with another and the same state at different times. Statistics therefore does not stand still. This recommendation was first formulated by Leibniz in a manuscript of 1680 (Sheynin 1977, p. 224). O. S.

Part 2. The Boundaries and the Benefit of Statistics

26. Knowledge ought to be subdivided

The field of human knowledge is unmeasurable and the human mind, even when applying its highest possible efforts is unable to embrace it, so it ought to restrict itself and study that field by parts. After combining a certain number of kindred knowledge under a single general idea it considers such a combination as an independent whole, as a specific science and separates it from all the rest.

The experience of three centuries proved that such a division of labour especially fosters the success of mankind and that the perfection of knowledge mostly depends on this condition. Each science considered by itself is always more or less connected with other sciences, but the philosophical mind incessantly attempts to extend the scope of the separate sciences and restore [or reveal] its internal connections with other sciences. However, we should nevertheless try to define exactly the boundaries of each to prevent confusion and inconsistences in the notions which are so harmful for the success of sciences.

The boundaries of statistics as compared with those of other branches of knowledge are clearly seen in its definition but it is not superfluous to study its distinctive nature more precisely and the more so since its independence had been formerly questioned[1].

Notes

1. See however Note 1 to § 13. O. S.

27. Similarity of statistics and political sciences and their separation

The subject of statistics is the state and it therefore ought to be connected necessarily with all those sciences which have the same subject, i. e., with political sciences. For precisely explaining the relations between statistics and those sciences it is necessary to show their scope or content.

The state is a moral organism. It is living, organically developing in space and time. A reasonable life develops in conformity with its aim. So how had it developed, what it is now and what will and should it be in the future?

If security and welfare constitute the main aim of the entire government activity, then the scope of political sciences includes all the knowledge which enables us to grasp how to attain that aim in the best way and how was it attained previously and is attained actually by the previously existed and nowadays existing states.

Therefore, according to Schlözer, in general the entire field of political sciences is subdivided into philosophical and historical sciences. This subdivision is not sufficient because some political sciences can only become systematic when philosophical rules are combined with historical facts. Indeed, political sciences are those which teach us how states under given conditions can become such as they should be. It follows that political sciences cannot be either purely philosophical or purely historical and therefore constitute a separate class. And so, political sciences are subdivided into three classes of the sciences of the state:

Philosophical. Here are the public and common law. Both are subdivided into philosophical or general and positive depending on whether they issue from reason or the existing established order of a state or of several states[1].

Philosophical-historic. Here we have politics, i. e., the science about the best way to arrange a state and manage it. Its parts are concerned with the internal structure and external relations of the state respectively. The former consists of *the science of the measures of state security and organisation* (Polizei-Wissenschaft), the latter is diplomacy.

Historical. These include political history and statistics.

Note

1. This, then, is an explanation of a positive science. O. S.

28. Statistics and the philosophical public and common law

Statistics is distinguished from the philosophical public and common law. Indeed, the latter issues from reasoning and their subjects are notions whereas the former borrows its materials from reality and experience. However, there also exists a connection between them, distinct but important. Many data borrowed by statistics from the positive public law can only become understandable and clear by those philosophical laws, for example, from the doctrine on the succession to the throne.

29. Statistics and the positive public law

Because of its historical direction the latter has a direct similarity with the former. At the beginning of the independent cultivation of statistics the positive public law only seemed to be its part since statistics borrowed very much from it. At that time, some authors complained that statistics had done away with the independence of that law and consequently some contributors of statistical writings attempted to banish it completely from statistics. However, after considering how greatly some rules about the mutual relations between government and subjects influence the attainment of the aim of the state, we ought to agree that a representation of a state will be incomplete and imperfect if these features are missed. In the practical European law a statistician will also find many indications which are necessary for a complete understanding of the external life of a state.

And so, statistics ought to borrow some objects from the positive public law although only those which have relations with its goal. It is exactly this goal which constitutes the essential difference between both sciences. The positive public law simply sets forth its notions without considering the results of their practical application. Statistics, however, is only studying that, which, belonging to that law, influences the aim of the state. In this sense statistics can never avoid public law, but the borrowing mentioned above does not at all harm the latter's independence since the borrowed is only its part.

30. Statistics and politics

The connections between the two are much closer than between statistics and the science of laws. It is situated in the middle between general politics and practical politics or the art of governing. Furthermore, the distinction between statistics and politics is also easily seen from their very essence. Politics considers measures for improving security and increasing welfare whereas statistics shows reality and to what extent is that aim indeed attained. Politics studies the methods of increasing the public wealth whereas statistics is only investigating the existing. Politics attempts to preserve and improve the external relations of the state whereas statistics compares the merits of the state and other states and its relations with others.

However, in spite of the distinction between the general ideas of those two sciences they are closely connected. Politics uses statistical remarks for explaining its rules but does not consider the time to which these borrowed statistical data belong[1]. For a deep understanding of statistics we need the knowledge of politics and the statistician must arrange his objects so that his science will be able to answer all the questions of politics and political history. Just the same, politics cannot become perfect or thorough without statistics.

Note

1. Remarks had somehow become data. O. S.

31. Statistics and the general political history

Statistics had been often confused with history, namely, when the indications of the distinctive features of both were confused. Nevertheless, all authors had agreed that these sciences are different and indeed, even their subjects differ. History describes man-made remarkable features in any territory whereas statistics only describes remarkable features of the state [not only man-made!] so that it only deals with states. Then, history describes events and coups d'état whereas the subjects of statistics are the components of the state. History deals with any time period, statistics only considers one moment.

When authors reason about history from the political angle, as did Schlözer, Schpittler, Johann Miller, Geren (Герен), Wachler, Salfeld (Зальфельд), Rotteck, Lüder, Pölitz and others, the distinction between the two sciences becomes difficult. History, thus considered, explains not only the internal, necessary connections of cause and effect between events, i. e., pragmatically, but the conditions of the internal and external life of previous and present states. In this respect statistics is very close to history since its subject is also the explanation of the internal and external life of states[1].

Many authors looked for the distinction between history and statistics in that the latter only describes the present conditions of states whereas the former pictures such events which show how a state had passed all its previous conditions up to the present. Nevertheless, such a distinction is superficial since it would have followed that, on the one hand, statistics is impossible because time is incessantly going on, and, on the other hand, that history is a collection of statistics because in the old days each past time was present.

It will then be even possible to say that a dated statistics becomes history and that statistics is a part of history and Toze's definition will be true: *Statistics is recent history*.

Much more thorough is the distinction between pragmatic history and statistics which allows that statistics, since it describes states as they really are, should not restrict its efforts only by present events, but, without considering the time, can include data which follow from remote events if only they influence the present aim of the state. Then statistics will describe not what had been occurring *successively*, but what exists in the state *now*. Without describing past or present events it is then satisfied by considering their results which influence the achievement of the aim of the state.

It cannot be denied that the modernity of the described objects only lasts for a moment since everything changes with time so that, as it seems, statistics cannot compile an enclosed whole. However, we said in the definition that a statistician describes exactly such a moment, considers it as present and completely isolates it from the past and future. It follows that the Shlözer formula which I mentioned in § 25, *History is statistics flowing and statistics is history standing still,* not quite satisfactorily separates those sciences and weakly expresses their relations to each other.

It is much better to say that history relates to statistics as poetry to painting. The last mentioned can only represent an action at a certain moment whereas the former bravely hovers and dares to describe not only the present but the past and the future as well. In spite of the distinction of these sciences in that their main ideas and aims are different, it is impossible to say that one of them can do without the other. Statistics often seeks help from history, and history often needs statistical remarks. At the same time none of them can yield its materials to the other without essentially changing its properties. In statistics, historical facts become statistical and vice versa.

Note

1. This is an important statement. See however § 54. O. S.

32. Similarity between statistics and descriptive and historical sciences and its distinction from them

Thus, statistics is a descriptive historical science and it should therefore be similar to all these sciences which consider and describe territories with their produce and man himself. For this reason statistics had been combined with **geography** because it borrows many objects from it. Statistics includes the entire political geography. Description of the situation, size, climate and soils of states and many other objects are borrowed from mathematical (?) and physical geography. Indeed, geography provides statistics with many important materials, especially when it expresses the main forces and statistics cannot be studied before geography. At the same time statistics is not a branch of geography and essentially differs from it: its subject is the state whereas for geography it is the Earth.

Statistics can only exist for countries which are territories of states, but geography studies any country. The essential difference however consists in that the former, although it borrows geographical objects, considers them from another higher point of view and explicates them in a different way in relation to the aim of the state. Thus, if geography indicates that Great Britain is an island, statistics, after borrowing this fact, represents it as the basis of the might of this country[1]. It follows that statistics differs from geography as much as from any other science which has [is based on] another main idea.

In the same way statistics differs from **topography** which is a part of geography and describes a country in the smallest details[2]. Statistics relates to topography just as to geography. The confusion of geographical, topographical and statistical notions in the so-called descriptions of countries, just as any other confusion of heterogeneous knowledge, can be unfavourable for the success of these sciences if it becomes widespread. Only a study of those separate sciences can guarantee success.

Ethnography also differs from statistics. It is a description of various nations and tribes according to their geographical dissemination and character. The subject of ethnography is a nation of the same origin and language[3] and it therefore follows a nation in different and most remote countries. If, for example, it describes the inhabitants of Graubunden [a canton in Switzerland] and Vlachs of Transylvania and Turkey as a nation of common origin and language it does not think whether this nation is living in the same state or in many states. If otherwise, then it is only to show the influence of civil life on the character of the nations and on the change of its natural properties.

Statistics deals quite differently. The state is its main thought. Cimbri, the inhabitants of Wales, Caledonians, who live in northern Scotland, and the English are described as a single nation since they belong to the same state.

Notes

1. Obodovsky borrowed this example from Schlözer (1804, § 14, Item 3). O. S.

2. Topography is the geographical and geometrical study of a locality. Topographical maps are compiled to large scales. O. S.

3. Nowadays ethnography is understood as a science of people and culture. O. S.

33. The benefits provided by statistics

It seems that such a question concerning any science should not be asked since we ought to like unselfishly any truth and therefore any system of truths as well. However, bearing primarily in mind the perfection of his own mind, the student of a science can imagine something ideal as material and therefore weigh the practical benefit of that science.

A government dignitary, an official in the supreme circle of state service and each citizen, – all of them need statistics. The dignitary with an ardent zeal for his fatherland, deep knowledge of theoretical politics and unusual mind but without statistical knowledge will not be useful for the state and can even be harmful. Just so the most skilful physician who did not carefully consider the condition of the patient is useless and possibly harmful for him. Let someone say that experience, reports of offices, protocols, official evidence can better guide a dignitary than statistics. But will not his mind be refined by statistical knowledge and become able to apply duly the sources which are thus opened for him?

Statistics is especially needed by diplomats. A correct estimate of forces of his own and foreign states, a sure view of the mutual interests of states, an exact knowledge of the established order of the state determine political relations and even the measures of internal management, for example, national economy, finance, military force. A wrong viewpoint on these objects of superior state management can be pernicious.

Statistics is both necessary and useful for officials who are moving in the circle of the superior state activities not only for their own advancement but also because without statistical information, possibly apart from mechanical clerky work, they will be totally or partly worthless.

If some state manager does not feel the spirit which animates the government and does not see the connection of his field of work with its entire activity then even the most proper and wisest measures adopted by the highest authorities often cannot be duly realized. What can an excellent master achieve without skilful assistants and tools? Can an official, who is directly connected with the people and on whose reports important state measures are sometimes based, duly describe the conditions of the studied objects if he is unable to view them from a proper angle?

Finally, each citizen needs statistics which is useful for him. It nourishes his patriotism and preserves his national character, these inexhaustible sources of civil virtue and heroic sacrifice of himself for the defence of throne and fatherland. Each educated citizen wishes to know what is going on in the state. The question: what new events are occurring? is always on the [educated] citizen's lips. It proves his participation in public affairs and is closely linked with his affection for the sovereign and fatherland, but often leads to absurd delusions, false trends, harmful opinions if only their alarming current is not quenched by open and well-founded statistical information.

Part 3. The System of Statistics

34. The notion of system and its necessity

A system of statistics is the totality of duly arranged objects belonging to the knowledge of a state or many states. Scattered remarks about separate statistical objects or their unplanned combination cannot constitute a science. The property of science requires a strict order of its parts and their incessant connection which depends on the main idea.

The necessity of a system is based on the general striving of the spirit for unity. To satisfy it is the more necessary the more does the mass of our knowledge increase and the more are we convinced in that our knowledge becomes thorough and clear due to its logical unity.

35. Statisticians do not agree about the [required] system

The system of statistics depends on the definition [of this science]. There exists a necessary and tight connection between the latter and the parts of statistics. If the definition is correct then the separation of statistics into parts is also true[1] and vice versa. When the definition of the parts of statistics is true, it becomes easy to imagine its true definition.

Consequently, the authors, who disagree about the definition of statistics, cannot have one and the same system of this science. Indeed, almost each statistician keeps to his own order and is even guided by distinct plans when describing various states or the same state at different moments. One of them admits as an essential part of statistics what another statistician unconditionally rejects[2].

Notes

1. This is doubtful. O. S.

2. Thus, Schlözer had totally banished geography from statistics. [In 1804, in § 8, he touched on this point. O. S.] Donnant thought that it was a branch of statistics; Lucka decided that a geographical description of territories should even be a main part of statistics; Clament stated that statistics and geography are absolutely different just like a deep investigation of an object differs from its superficial study. According to Mannert geography is the assistant, the mother and sister of statistics. Some authors considered topography the daughter of special geography, of another branch of statistics (Clament and French statisticians) whereas the German authors banished topography from statistics. True, Schummel (Шуммель) ardently defends the opposite view.

[Obodovsky continues to discuss the disagreement among the authors:]

Sprengel thought that the description of national character was difficult and therefore unnecessary and moreover that it is seen in the way of life, amusements etc., but Meusel and many other authors described it. Schlözer attached unusual significance to the national character. He thought of precisely, numerically defining how diligent are people, how nimble and strong they are. [There is nothing of the sort in Schlözer (1804). O. S.] A. O.

36. The most important systems of statistics

However, since the notions of statistics of various authors are in many respects similar, so some similarity is also seen in the subdivision of statistics. Three systems, those of Schlözer, Niemann and Hassel (Gassel) can serve as prototypes whereas all other statisticians only differed from one of them by some nuances.

1. The Schlözer system is based on the formula *vires, unitae, agunt*. The first word signifies the main forces (the people, the land, the produce and the money in circulation). The second one, of combining those forces (the regime of the state [monarchy, republic etc.]) and the established order of the state, and the last word means the actual use of those forces, i. e., the management of the state.

2. The Niemann system. It combines the statistical data in two parts:

2.1. The statistical description of the land or territory belonging to the state (its origin and combination of its parts, their interconnections, the ability of fertilizing the soil, the inhabitants).

1) Historical description (the components, the tribes of population).

2) Geographical description (size of territory, boundaries, political subdivision, number of inhabitants).

3) Physical description (the kind of surface, climate, produce, inhabitants).

2.2. Statecraft

A. Statecraft proper

1) Established order of the state (the established order of the state proper, civil, church, educational established orders)

2) Management of government

a) Organisation of the legislative and executive authorities

b) Legislation and its administration or the description of the acting government institutions

c) Political statecraft

B. The science of the people (Nationalkunde)

1) The study of the industry and national economy (cultivation of land, raw materials, manufactures, trade). The components of the people's property (forces (?), immovable property, cattle-breeding, money), welfare

2) National character and enlightenment

3. Hassel (Gassel) (1822b) describes the state with regard to physical forces, then how and by which method does it act. He also subdivides the statistical data in two parts

A. Elements of the main might

The location, boundaries, size, components of states. Inhabitants. The extent of cultivation of the land, produce. Technical diligence, trade. Enlightenment. Finances of the state, military might.

B. Elememts of political life

a) Established order of the state. Main laws. Regime of the state. Monarch and his house. Established civil order.

b) Management of the state

c) Political relations with other countries[1].

Note

1. Here are examples of the deviations from those forms (?).

Donnant (1876) subdivides statistics into analytic (everything about the balance of various states in some parts of the world); particular (the study of topography). He also considers properties (physical and moral sources of the might of a state) and internal statistics (it deals with both particular and general facts and distinguishes each part of a vast state).

Gatterer and Toze subdivide the objects of statistics into four sections (and Remer into five sections): geographical and natural conditions of the state, its civil and church established orders, the condition of erudition and enlightenment, political relations.

Lüder, in his Introduction, numbers almost a hundred sections placed under 80 categories, very thoroughly but without any discussion or systematic order.

De Lucka (1796) placed in his Introduction those objects which, in contributions of other authors, determine the content of statistics. He surveys physical and moral forces (and reckons among the latter the regime, the established order and management of the state). He calls all the rest statistics proper (statistics of the police, of politics, trade, finance, clerical work, state power).

Malchus follows Schlözer with some changes. His sections are 1) *the sources of the main forces* (the land, natural yield, inhabitants); 2) *elements of the wealth of the state*, industry; 3) *the results of using the power of the sources and elements*, national wealth etc. 4) *established order of the state*; 5) *the regime and the management of the state*; 6) *political relations with other states*.

It is not amiss to add Fourier (1823/1834, pp. VII – XII) the more so since Obodovsky only mentions German sources. Fourier lists in detail the following objects of statistical research: **1)** Territory (physical and political structure. **2)** Population (general condition, movement, cities, artificial structures, trades, condition of work). **3)** Civil institutions (government, administration, judicial and religious institutions, institutions of public assistance). **4)** Military might. **5)** Industry and commerce. **6)** Finances. O. S.

37. Criticism of the systems of statistics

The Schlözer system is undoubtedly better than the others since he clearly separates the statistical objects from each other according to their properties and arranges them in a manner in which they appear in a tighter connection as cause and effect, as the condition and the means for its fulfilment. But it is impossible not to agree that his first section contains too heterogeneous objects and is too extensive and arbitrary. And the deep investigations of Adam Smith had proved that the money in circulation in a state cannot be considered among the principal forces.

The main defect of all the described systems and of many others is that the principal parts of statistics are determined by some main objects of the state, and that those parts are subdivided in the same way. Three statistics are usually named: those of the state, of the people and government. They are subdivided into innumerably many other statistics, each being a separate whole. Thus, the statistics of the state is separated into statistics of agriculture, wine making, cattle breeding, hunting, fishing. Then, statistics of the people means statistics of their physical and moral strength. Statistics of the government means statistics of the police, of jurisprudence, military forces etc[1].

The imperfection of such statistical systems is clearly perceived: statistics as a science seems to be a collection or compilation of heterogeneous knowledge. Take any statistical contribution written according to such a system, forget its title and try to determine: to which science belong its sections? Even the most knowledgeable statistician will experience difficulties in deciding whether to statistics or to any other science. Indeed, he will find there fragments of physical geography, ethnography, commerce, technology etc.

True, statistics, like other sciences, gets materials from various sources and necessarily deals with them in its own way so that it is seen at a glance that they belong to statistics. If, in a certain contribution, we see something contrary, we ought to doubt that a true system of statistics is present there.

Note

1. The grammatical construction of the Russian phrase was wrong and the translation is only probable. O. S.

38. The reason why systems of statistics are imperfect and the means to overcome it

So what is the reason for the failure of most systems of statistics? We may surely answer: any statistical contribution fails if statistics is there subdivided according to material objects. There are innumerably variable objects and unity in their subdivision cannot be achieved. How absurd it is to subdivide philosophy according to the objects of the external world since any such objects can be studied by that science. And it is equally absurd to subdivide statistics according to the immeasurably many variable objects belonging to a state.

Let someone say that statistics is an experimental science! It proves nothing since the conclusions which follow belong not to a science but to those objects which are certainly subjected to rapid changes. And so, the parts of statistics cannot be determined according to the variable objects of the external world. We ought to search for its subdivision in the field of the mind whose knowledge is distinguished by strict unity and necessity of order. If statistics should find out to what extent the aim of the state has been actually achieved, it should determine beforehand those means which secure the achievement of that aim in general and in a systematic scientific way determine their *internal and external* nature. If successful, that study will arrange the necessary means for attaining the aim of the state in an entirely systematic way. Only then it will be possible to consider and study the great variety of the objects to find the general concealed in them if only our observations are faultless and attention is paid to all the essential and heterogeneous.

Only thus we can keep to the true path and save for statistics the merit of a science. Otherwise its study will not be attractive for a philosophical mind and, furthermore, impossible in its entirety. The authors of theories, Niemann, Zizius, Klotz, Pölitz, Koch-Sternfeld, Holzgethan and the practical statistician Schubert had applied that proper method.

39. Statistics of the internal and external relations of a state

When considering a state, a political body, from that viewpoint, it should be presented according to its internal and external relations. A man can be studied all by himself and in relation to others, so also a state, all by itself or in relations to other states under whose influence it is changing.

The internal life of a man determines his external existence, and the external relations of a state depend on its internal circumstances so that consideration of the external relations should therefore be secondary. True, history shows that the external conditions sometimes completely change the internal life of a state and we may therefore think that the internal life depends on its external life as well so that the latter is more important than the former. However, thorough observations will convince us that, although the external circumstances influence the state, their result depends on internal conditions. And we may surely maintain that a statistical study of the internal conditions of a state is more important than the investigation of the external conditions. Therefore, statistics is subdivided into a representation of both its internal and external conditions.

40. The internal conditions. The main forces

When turning our attention to the internal condition of a state we easily see the subsequent division of the objects belonging to its very aim. If security and welfare are those boons for whose sake people unite into states, then the origin and the life of a state depend on the existence of its forces and abilities. Without them it is impossible to imagine any action so that the aim of the state is never achieved.

These forces and abilities consist in the territory of the state and its inhabitants. The land and the people are therefore the main forces and their existence is an essential condition for any activities of the state (*conditio sine qua non*).

The land. Even a superficial consideration of the land, the region of the state, the territory, shows that it is the sum of the abilities and forces and that it essentially influences the achievement of the aim of the state.

The location of the land (is it an island, maritime or intercontinental) essentially influences the development of the state. When parcelled out or encircled by foreign lands or of a small size it harms independence. If the territory is too large the speed of [the realization of] government measures sometimes lowers. Natural boundaries provide more measures for repulsing external enemies whereas artificial strengthening of borders is greatly expensive.

Here, scorching heat or lethargic frost leads to the laziness of the inhabitants or weakens their intellectual faculties. There, on the contrary, a happy combination of heat and frost develops those faculties and makes the inhabitants industrious. Here, mountains assist fruitfulness (?), there, their lack hinders it. In a certain state a happy system of rivers connects the remotest localities and fosters the sale of the produce and an increase of production. Elsewhere, rivers are scarce and the most excellent gifts of nature become useless and perish. Here, the barrenness of the soil makes all the efforts of diligence futile, elsewhere excessive fertility lulls the strength of man.

After considering these phenomena, who will doubt that the land of a state influences all its life and all its manifestations in the political world. And we may say that a statistical representation of a state cannot be complete if it does not discuss the forces and methods which the land is providing. Such a representation ought to be directly included in the statistics.

The people. The people which constitute the state foster the achievement of its aim in different ways and the following considerations show how to consider the people in statistics.

1. The increase or decrease of the population is a most important indication of the change of welfare and security of the state. Indeed, each citizen can assist the achievement of that aim, i. e., to help to foster security and welfare either by defending the state from external or internal enemies[1] or by increasing its wealth by his labour as a farmer, artisan, manufacturer, merchant, or by paying various duties and taxes. And the more there are such useful citizens the more secure and prosperous the state ought to be. Statistics therefore requires the knowledge of *the entire number of inhabitants*.

2. In some states the distinction of the inhabitants by their origin and language disturbs the unity of one of their main capabilities. However, we should consider the ratio of the numbers of the governing people and of those of different tribes and on their (?) geographical distribution. In the Russian Empire the Russians greatly outnumber the members of all the national minorities[2] and moreover they are living in the middle part of the country and thus beneficially united. At the same time the people of other origin are living at the edges of the Empire, their number is small and they are separated by geographical position and languages.

On the contrary, in the Austrian Empire [1804 – 1867] many peoples of different tribes are living in large numbers side by side and hamper government measures, especially legislation and administration of justice, by differences of characters and languages. This latest example shows that in some states *the separation of the inhabitants by origin and language* largely influences the attainment of the aims of the state and should be shown in its statistical representation.

3. Religion of the people is even more important than origin and language since it touches the inner life of man and, furthermore, contains the education of most. The history of

Western Europe from the mid-15th to the beginning of the 18th century shows how great had been shaken many European states by religious hostility. Tolerance has since achieved essential success, but religious differences will always influence the aim of the state and statistics ought to show *the division of the inhabitants by faith and religion*[3].

4. The development of the political life necessarily leads to the creation of different classes of citizens. The most ancient hereditary difference had been between the free citizens and slaves [not a class of citizen!] which easily shows the main features of later lifelong gentry and peasants. Then there came the honoured gentry, bestowed on some, often hereditarily, by the supreme authorities as a prise.

In the Middle Ages there appeared between the gentry and peasantry a third hereditary class: the citizenry or the middle class.

During the crusades the higher estate acquired a yearning for a comfortable life and luxury and many peasants became artisans and later traders. Their income soon made them independent from their masters, they united into special settlements encircled them by walls and ramparts to defend themselves against so often predatory attacks and began to be called citizens. Their wealth gradually increased and they became greatly influential.

Apart from these three hereditary classes which became an essential part of the population many personally titled people had appeared after someone filled a post which was later recognized as important and necessary. This happened first of all with mentors in the truths of the Christian religion who formed the clergy. When a standing army was formed, a military estate had emerged; in addition, the branches of the state management multiplied and a status of civil officers came into being.

All the government estates are divided according to the aim of the state into two classes, productive and unproductive. The first includes farmers, artisans, manufacturers and merchants [as mentioned above]. The second consists of civil officers, military men, clergy and scientists [see however below].The former ought to procure all which is necessary for the life of a state and increase public wealth, it largely assists in obtaining material comfort which is the foundation of the highest development of the aesthetic, mental and moral. The latter should provide peace of mind, security and [teach] all the methods of producing the necessary, useful and pleasurable for life. The totality of such things is indeed the public wealth.

The balance of all the estates and classes of the state is an important condition for achieving its aim and statistics ought to include a section on *estates* and the ratios of their numerical strength. Statistics can also include the number of inhabitants of towns and rural areas which assists in finding out what kind of industries is prevalent in the state.

5. Now we can easily convince ourselves in that the density of population of different states very much differs. In England without Wales there are more than 5400 people per square mile, in Germany, more than 3300, in Spain not more than 1650, in European Turkey, 950 and in Sweden and Norway only 290.

It is easy to understand that that difference depends in some cases on the quality of the climate and soil, in other states with good climate and soil, on the extent of the enlightenment of the population, and measures of the government. The more secure are the rights of citizen, the more sources for industry there are, the more thorough is the upbringing of the population and the better are the moral relations in the families, the more properly does the population increase.

A gathering of large numbers of people in a small region increases its needs and improves the means for better and easier satisfying it. Information about the income had been compiled when the income tax was introduced in England, in France on the occasion of adjusting taxes and similar materials were collected in other countries. They sufficiently convince us in that the *density of population* is an important statistical subject.

6. When describing a population of a state statistics includes many other data very fruitful for finding out to what extent the aim of the state was achieved. Thus, the number of families and therefore the mean number of their members. When that number is large, we may certainly say that people live moderately and frugally and that the moral is not corrupted[4].

The relative number of criminals shows the extent of the morality of the population. In a similar way the relative number of births and deaths is calculated as well as other indications (the attitude of the population towards marriage, the ages, the number of men able to carry arms etc.) which are collected in *tables of population*[5].

Notes

1. During long periods of time Russian authorities had been attempting to overcome its internal enemies: the terrorists (which appeared after 1839) and progressively minded citizens, students in the first place. O. S.

2. This reasoning is superficial. First, not only Russians but Slavs (Ukrainians and White Russians as well). Second, Tatars and Bashkirs lived (and live) in the middle of the country. Third, religious faith, the recognition of the Russian Orthodox Church, was more important than nationality. O. S.

3. In Russia, the relative number of Muslims has been gradually increasing and nowadays they have to be most seriously reckoned with. O. S.

4. In 1823, in Paris, there were 659,172 [659.2 thousand] inhabitants and 224,922 [224.9 thousand] families, less than three persons per family. The worst indication! A. O.

5. He could have added: attitude […] towards inoculation (the not quite safe preventive measure against smallpox, practised until the introduction of the Jenner vaccination).

Ivanovsky (1890, pp. 124 – 132) properly remarked that both sanitary and criminal statistics are extremely important although the former barely existed. On p. 132 he maintained that in Russia the registration of criminality was better than in France (the cradle of criminal statistics!). O. S.

41. Internal conditions (continued). The structure of the state

The natural means for attaining the general aim of humanity[1] should be mutually adjusted and properly directed to the aim of the states. Otherwise none of the two aims will ever be attained. Therefore, the authority of a single person in a state is recognized. He sets in motion the main forces and abilities for achieving the aim of the state, removes all obstacles to the lawful development of those forces and abilities, and when needed, turns to compulsion.

A state cannot exist without a government since only it connects all parts of the state into a single whole. These parts therefore interrelate as aim and means, as cause and effect. Only then an inner unity is occurring and the state becomes different from all other states and is an independent whole. Its structure is adjusted and, in a word, the state becomes an organic society.

The established order of the state. In various states the supreme authority is arranged in different ways. It ensures the means and conditions for attaining the general aim of the state in a civil society. The totality of all those means and conditions for attaining that aim, for the state to become a harmonious whole, is called the established order of the state.

The supreme power is vested in a single person or a collective person and thus the *form of governing* is determined. The former is called monarchy, the latter, polyarchy[2]. The supreme person has three branches of authority: legislative, judicial and executive. Legislation belongs to the monarch who can share it among some class of citizens, the representatives of the people. In polyarchies, or the so-called republics, legislation is in the hands of the most excellent people. This is the basis for separating the monarchy and polyarchy.

The management of the state. The single or collective person has the right to act for the achievement of the aim of the state. He reigns or governs. This means granting the laws for the subjects and arranging properly all the institutions applied by the supreme authority for carrying out its will and adjusting the laws for their applicability to all special cases. This indeed is the management of the state.

And so, the essence of this management is the setting in motion all the laws of the state. It therefore should extend over all the branches of legislation[3], *to internal administration of justice* (*the police, public economy, finance and military forces* [cf. end of § 37]). Statistics ought to consider *the management of the state* in all the mentioned directions.

Notes

1. In the social and political sense a single humanity never existed. O. S.

2. Aristotle and many later politicians distinguished three forms of governing: monarchy, aristocracy and democracy. But the last-mentioned form cannot exist since a society which constitutes a state cannot at the same time be governing and governed. Even in polyarchy the number of governing people ought to be restricted as much as possible since the difficulty of unity or agreement must increase with that number. If some change of the polyarchy is called democracy, the latter is really the governing of a few. A. O.

In 1619, Kepler quoted an author of a contribution of 1586 who had followed Aristotle and connected those forms with the harmonic, geometric and arithmetic proportions. Kepler, as it seems, was in favour of democracy, see Sheynin (1973, pp. 119 – 120). O. S.

3. They are mostly measures of security since, being safe, the people will attain welfare all by themselves. A. O.

Nonsense, suffice it to mention the system of taxation. O. S.

42. Internal conditions (continued). Culture

And so, the land and the people constitute the abilities and the forces granted by nature, whereas the structure of the state expresses the active condition of those abilities and forces, i.e., the drive to the aim of the state. Now, we have to consider how these forces and abilities have been developing and forming so that a reasonable will can easier direct them.

This problem leads us to the study of culture which we understand as the measure of the development and creation of all the physical and spiritual abilities and forces as well as the peculiarities of that development and creation. A culture is called *physical, technical, aesthetic, mental and moral-religious* depending on its belonging to the preservation of physical life with its comfort or to the action of the ability to feel and to find out and to the [boosting of] morality.

43. The influence of the culture on the natural abilities and on the established order of the state

Since culture influences natural abilities, they completely change. It also provides new, previously unknown abilities and forces. Schlözer justly says that [Obodovsky enthusiastically quotes Schlözer's description of the transformation (actually, of destruction) of nature. Schlözer (1804) apparently does not contain that passage.]

Enlightenment changes nature and it shows us phenomena which nature would have never revealed all by itself. So also a state is the product of mankind ripened for absorbing the culture of mankind and it, the state, can only blossom when taking into account the necessities of culture.

Thus, enlightenment influences the natural abilities and forces of the state and the legislation itself takes into account these necessities. The land and the people represent forces, the established order of the state, its will whereas culture is the connection between force and will, it shows the direction to the aim of the state. Culture, therefore, is one of the most important subject of statistics.

44. The external conditions. The political position

The wider is the culture extending and, together with that, the more the population is growing, the tighter become the states one to another. At present, all the European states are interconnected and mutually act and counteract. No state can keep away from the chain that binds them or separate itself from the influence of other states, or, following its own arbitrary choice, independently adopt its system of national arrangement.

In this general connection of states each is more or less active or passive, more or less essential, and occupies a certain *political position* in the sequel of the other states. Statistics aspires to determine that position by the internal and external relations of the state taken in totality.

45. The interest of the state and independence

Any state has a common purpose with many others but it also aspires to attain a special aim of its existence which is destined by its natural or acquired abilities, location, occupation of its citizens, natural fruitfulness of soil, by the number of its inhabitants, degree of their enlightenment, etc. That special aim is its *interest*. Since various states have different interests it is quite natural that, while striving for their attainment, they ought frequently to clash hostilely.

We should justify the aspiration of any state to achieve its aim if only it keeps within the boundaries of the laws of rights, i. e., does not interfere with similar aspirations of other states. However, a state has no external motive for stopping at the boundaries of that law and nothing prevents it from continuing its policy without oppressing other states[1]. Each state should be therefore arranged in such a manner which impedes any other state to insult it or to violate some of its special rights. A state ought to be in such a position that other states will be unable to oppress it without expecting serious disadvantages.

Thus *independence* is achieved which ensures the possibility of striving for its aim without any hindrances. The extent of this independence is seen in its political extent [extent of political power] and weak states defend themselves by a system of political equilibrium, by agreements.

Note

1. The boundaries of the law of rights are still recognized! O. S.

46. Agreements between states

The political superiority of a stronger state can be harmful for a weaker body. To compensate this situation each weaker state should endeavour to connect with other states which will prevent the stronger state from depriving it of its independence and restrain any attempts to prevail over it. Thus occurs *a system of political equilibrium*.

To initiate such a system and at the same time to establish, continue and strengthen friendly relations and mutual connections between states *agreements* are needed. They stipulate that both sides cede each other some of their rights and unite for attaining a definite aim, whether an improvement of their relations or defence against violation of their rights (or against threats to violate them) or against both. Representatives of the nations or *envoys* are then needed for supporting such connections and testifying about friendly relations.

47. A survey of the main articles of a statistical representation of states

A natural and unconstrained order of a statistical representation of the parts and subjects of an entire science follow from §§ 39 – 46.

I. Internal conditions

A. Main forces

1. The land or the region of the state

α) The territory

a) location, form, boundaries

b) size

c) kind of surface (mountains, plains)

β) natural conditions for the development of the main forces

d) waters

e) climate

f) soil

γ) natural produce

g) mineral kingdom

h) plant kingdom

i) animal kingdom

2. The people

a) total number and its subdivision

1) by origin and language

2) by faith and religion

3) by estate (noblemen, clergy, citizens, peasants. Productive and unproductive classes. Inhabitants of towns and rural areas)

Note. The rights of those estates are shown in the civil order of the state.

b) relative number of inhabitants (their density)

c) tables of population (relative numbers of marriages, families, ages, births, deaths etc.)

B. The structure of the state

1) Established order

a) main laws of the state (general, civil, church)

b) form of governing α) for unrestricted monarchy

αα) monarch and his house, succession to the throne, symbolic indication of the might of the monarch (title, national emblem, courtiers). For β) restricted monarchy, additionally

ββ) the representation of the people or the estates who participate in the legislation. For γ) polyarchy, whether

αα) aristocracy, or

ββ) democracy

2. Management

a) general notion of the executive authorities αα) are the regions of the state managed separately or is the management centralised

ββ) how many ministries? Their interrelations

γγ) is there a state council and its duties; is there a controlling establishment

b) ministries separately and highest, middle and lower offices

C. Culture

1. Physical

a) agriculture in all of its branches (cattle breeding, silkworm breeding, bee-keeping, etc., hunting and fishing)

b) mining and salt-mining

2. Technical

a) factories and manufactures

b) commerce

3. Aesthetical. The condition of fine arts and their establishments

4. Mental. The condition of educational institutions of higher and lower, general and special education, scientific societies. The condition of literature

5. Moral-religious. The representation of the moral qualities of the population, the condition of the enlightenment, religious conditions of the population, tolerance, fanaticism etc.

II. External conditions

1. Political extent [extent of political power], **relations with other states**

2. Special interest of the state

3. Agreements

Part IV. The Methods of Statistics

The previous part shows the content of statistics. Now we ought to show how a statistician can assist the success of his science, i. e., to show the method of collecting statistical data and explaining them.

48. The method of collecting statistical data or the sources of statistics

The sources of statistics can be

1) state documents, law codes, peace treatises, trade agreements, conventions, reports of the ministries, journals and registers published by the government, censuses of population, charters, privileges

2) privately published journals, travels [travelogues], topographies

3) oral information from knowledgeable and impartial people

4) one's own observations and studies

Statistical criticism estimates all these sources according to their external and internal worth. The latter is determined by the quality of the authors and circumstances, and the former, by the quality of the sources themselves. Private information should be especially criticized, but some official documents are not exempt from criticism either if based on doubtful indications.

Finally, much depends on the quality and properties of the object itself. Thus, agricultural tables are more trustworthy than tables of manufactures which in turn deserve more trust than commercial tables. Best of all for approaching truth is to base oneself on official documents and compare all the other sources with them.

A statistician who applies various sources encounters many difficulties:

1. Many statistical sources are not printed and can only be obtained with difficulties. Some other sources, although printed, are not included in the general book sale business, or are too voluminous, expensive and often even incomplete.

2. When collecting statistical materials for a general statistics you have to master many foreign languages and furthermore perfectly understand the language of business. Schlözer [1804, § 24] remarked:

A man can read Voltaire [in French] *but still in many places of a* [French] *instruction in finance or manufacture he will be helpless even with best dictionaries*.

3. Suppose that a statistician has all the possible materials, but it is impossible to imagine that one man without help from others can duly order them without large expenses, without victimizing himself and all his time. Statistical criticism ought to be lenient to statistical contributions and especially to numerical details contained there[1].

Note

1. The atmosphere in the history of mathematics is nowadays charged with universal leniency (Sheynin 2018). O. S.

49. The method of providing information about statistical knowledge

That method can be either descriptive or analytic. The former, a detailed and clear portrayal of the really existing state, can be *ethnographic, comparative, tabular or linear*; alternatively, *factual or pragmatic*.

50. The ethnographic method

Here, each state is described separately according to an adopted system. This was the method of Achenwall, Remer, Meusel, Sprengler, Mannert, Millibiller, Krome, Hassel (Gassel) and many others, and this is also the method mostly applied in German universities. Schlözer [§ 23 bis, Item 4] called the statistical description of states by the ethnographic method *German university statistics*[1].

This method has unquestionable advantage in providing a complete and clear notion of each state: the attention is only turned to one object. However, it, that method, assumes that all the data necessary for completeness are thoroughly collected and, moreover, that there is enough time for supplying detailed information [to the listeners] which is needed for that method to be appropriately useful.

However, the extensiveness inherent in that method is mostly very disadvantageous and scientists had therefore attempted to remedy the situation by various means. Some of them decreased the number of the described states and represented the selected states [even] more completely, but their choice was unfortunate: they paid most attention to states which were at the time politically prevalent. They did not take into consideration that in smaller states the moral and civil life was often developing more purely and stronger just because the forces in those states were concentrated[2]. Such states certainly deserve preferential attention for cognition of the elements of science.

Other authors avoided that mistake but had usually forgotten their own fatherlands and bordering countries in spite of the doubtless preference of national statistics.

If, however, statisticians described all the existing states they usually restricted their study by showing military and financial power, its influence on world trade and the political might of states without bothering about the relations of their internal life.

Notes

1. In English, university statistics is another and better known name for statecraft. O. S.

2. This explanation is certainly insufficient. O. S.

51. The comparative method

Those defects of the ethnographic method led a small number of authors to the *comparative* method which is also known under the name of its inventor, Büsching[1]. It shows the statistics of various states simultaneously, by ordered totalities of main data which describe objects one after another. The similarities of, and the differences between states are thus explained.

We ought to agree that it is impossible to imagine that a complete separate mental picture of each state thus emerges. Just as in history, the comparative method cannot explain the individuality of the states. However, the Büsching method has its own advantages. Statistics of all states can be surveyed most promptly since many useless repetitions can be avoided. In addition, the cognition itself of the states is more thorough when they are compared with each other. And our imagination then becomes unintentionally excited by thoughts about how is the aim of the state attained in this or that state under given means and conditions.

Again, this method allows us to select the most suitable and most preferable data. It indicates those data on which the better structure of the political organism is based [which determine the better …]. Finally, this method allows us to discuss in detail those data which are recognized as especially interesting for the listeners. Schlözer [1804, § 23 bis, Item 8] properly praised this method and we can only regret that just a few authors had applied it. From more remote authors I name Büsching [certainly!], Beausobre, and de Lucka, and from recent authors, Malthus and Schnabel.

Note

1. Leibniz should be mentioned, see Note 4 to § 25. O. S.

52. The tabular method

Statistical tables result when applying the tabular method. Their purpose is to facilitate the collection and comparison of statistical data as well as the formulation of the inferences. A statistical table represents either one object with all its details and comparisons or many objects and orders them side by side. And it only deals with such data which can be briefly represented without long explanations. It is therefore mostly restricted to indicating the size of a state, the quality of its soil, measure of enlightenment of the population, number of inhabitants and its density (subdividing them by origin, language, and faith), number of towns etc. In a word, almost restricted to numerically expressible data.

We may already say that this method is not sufficient since only a small number of data describing the [approach to the] achievement of the aim of the state can be thus expressed. Statistical knowledge expressed in numbers is very precise[1] and definite. However, who wishes to restrict all the science of statistics to a table will only see the state from the material side and miss the moral forces which provide definiteness and character to social relations[2].

And so, statistical tables are only useful in that they provide an easy survey of statistical data, assist memory and can be applied for systematic repetition of the studied. They are also the foundation for comparing states which can never be done thoroughly without numbers. Tables will never lead to the uselessness of studying statistics or, in other words, to harming such studies, just as historical tables do not deprive political history of independence[3].

Notes

1. Numbers can be erroneous and, in addition, a usual mistake occurs when a number is not duly understood. Thus, the number of inhabitants of a town can only be known approximately. O. S.

2. This is an important statement: *moral* qualitative data are also important, but Obodovsky had not discussed them. O. S.

3. Tabular statistics which originated with Anchersen (1741) could have been the intermediate link between words and numbers, but Achenwall (1752, Intro.) stated that he had experienced *a public attack* against the first edition of his book by Anchersen. *Tabular* statisticians had been scorned, called *Tabellenfabrikanten* and *slaves of tables* (Knies 1850, p. 23). In 1734 S. K. Kirillov compiled a tabular description of Russia but his manuscript was only published in 1831 (Ploshko & Eliseeva 1990, pp. 65 – 66). I have found (but not seen) another source: Golitsin (1807). O. S.

53. The linear method

Linear statistics originated from the tabular method and can be understood as a changed version of tabular statistics. Its essence consists of representing everything numerically expressible by lines, circles, squares etc. Playfair, an English scientist [Royston 1956; FitzPatrick 1960], had invented it to facilitate the study of statistical data for those with bad numerical memory. However, the Germans applied that method much earlier.

The linear and the tabular methods are justified in the same way and the same advantages and disadvantages are therefore inherent in both. They ensure only a notion about numerically expressed objects in a state and cannot at all replace systematic statistics, However, is the linear method really useful and does it save time, as the linear statisticians claim? We ought to resolutely answer negatively.

First, any success in science depends on work in the proper direction and such trifles [as circles, squares etc.] can only seem important to laymen whereas a thorough scientist despises them. Second, when applying a certain method, we still cannot avoid numbers since only throwing a glance on, let us say, squares, which represent a state, we can determine the comparative sizes of states but not the size of each. And should not we return to numbers for understanding clearly the ratio of the territories of some states? These sizes cannot be determined by charts or maps without a scale, so also the linear statistical table cannot provide a clear notion about anything although only such notions are really valued in science[1].

Note

1. See the modern opinion about the linear method: Schmid (1978). O. S.

54. Factual and pragmatic methods

These methods differ in that only the latter shows the causes and effects of statistical data. Statistics completely concludes its goal only by providing statistical data and in essence even excludes any other kind of description since (§ 22) data constitute the whole content of statistics. The notion of *datum* is independent from causes and effects; the entire purpose of statistics is to represent accurately all the means which are necessary to judge whether the aim of the state is being attained, and to what extent, or not.

Some authors, especially [active] at the time of Gatterer [1713], founded the so-called pragmatic or philosophical method of statistics by entangling considerations and historical indications in their statistical studies. The purpose of the pragmatic exposition of statistics consists in showing how a present situation had been generated by the previous period; or, what caused it. They thought thus to provide thoroughness to statistics which, as some authors believe, it does not possess when described purely historical[1].

However, after discovering that there still does not exist any complete history of any state, and that it was therefore impossible to explain duly, in all aspects, the entire totality of statistical data belonging to it, we will convince ourselves how difficult it is to compile a pragmatic statistics[2].

Happily, however, statistics by its essence can do without pragmatism. Indeed, however entertaining it is to know the real causes of some object, we can have a completely clear notion about it without such knowledge. Anyone can clearly imagine, for example, the inhabitants of a state, the power and the structure of its armies, without knowing how it all came about.

We cannot deny that statistics, just like any other science, becomes clearer by history, but it is not obscure without it since it, once more just like any other science, includes in itself its own light. Here, it seems appropriate to ask, should not a historical survey of the increase or a decrease of a state, and especially of its size and population, from the beginning to the studied moment, precede its statistical description?

The authors of statistical contributions disagree, but all historical doubtless belongs to history rather statistics whereas the subject of statistics is only the present[3]. However, those who begin to study statistics as an independent science should, but not always have a thorough knowledge of history and a historical survey can be doubtless useful for them. Some authors had indeed included such surveys, we name Hassel (Gassel), Pölitz, Demian, Wichmann (Вихман), and especially Schubert who masterly accommodated historical survey to almost all statistical data.

Notes

1. See Note 1 to § 22. O. S.

2. Why the unrealistic *all or nothing*? And the purpose of the pragmatic method is not at all restricted to showing a historical process, again see the same Note. O. S.

3. Obodovsky (end of § 18) quite properly maintained that previous moments understood as the present may also be studied. O. S.

55. The analytic method

There was a period when the main purpose of the authors of statistical contributions was the collection and hoarding as much as possible statistical data[1]. But how can we find out whether these data are statistical, do they connect themselves to form a single whole, and how this whole [if it exists] differs from other branches of knowledge? These questions were not then considered very important and only an introductory few pages were devoted to answering them.

Such authors usually stuck to the indications of experience and therefore considered the material part of statistics as their main subject. However, this empirical method [approach] was unable to conceal that the mass of the statistical data had increased unmeasurably[2] and that no efforts were able to unite them into a system. Accordingly, statistics, in spite of every endeavour and zeal of its authors, could have only been useful for a short time, and even the best contributions were forgotten yearly and almost monthly, just like calendars. Not surprisingly statistics became a target of mockery[3] and the number of its defenders incessantly decreased. But still, the need for statistical data had not lessened and the empirical method emerged victorious. It did not require large efforts when corrections of dated statistical information became necessary. Tables were replaced by tables, numbers piled on numbers and statistics almost became a soulless compilation.

Then came Schlözer. He studied the defects of the statistical method of his time and revealed its complete falsity for the world to see. All previous statisticians except Conring exposited statistics in a scant introduction and hurried to describe statistics of the states. Schlözer, however, acted otherwise, he represented the theory as the essential and main part of statistics and showed a specimen of its application. Excellent scientists followed him and it is now doubtless that only the Schlözer analytic method is the true approach which directly leads to the goal. Nowadays no one doubts that statistics is a science and that anyone who learned how duly to discern, estimate, collect and arrange statistical data can describe statistics when basing himself on its theory. It is not anymore possible to reproach statistics for the variability of its data since each datum is considered from its constant and invariable side.

The boundless mass of material statistics became accessible to the human mind. The merit of a scientific statistician is not anymore based on the knowledge of all the numbers characterising the statistics of some state but, additionally, on the thorough cognition of the theory of statistics and material cognition [cognition of material statistics] which should be entirely based on the theory, and, finally, on the ability of being able to create statistics if only materials and circumstances require it.

Notes

1. Biot (1855, pp. 1179 – 1180) opposed the publication of a great number of meteorological data useless for the general reader of scientific periodicals. O. S.

2. Cf. Lüder (1812, p. 9): the beginning of the century witnessed *legions* of new data. I adduce, however, the remark of Descartes (1637/1982, p. 63): *experiences* become the more necessary the more we advance in knowledge. O. S.

3. Lüder [1817, p. v] had railed against such authors, i. e., against empiricists, but he did not consider the theory of statistics. A. O.

He aimed at destroying statistics and (p. ix) likened it to astrology. Did he really have only empiricists in mind? Either bearing in mind this criticism or not, in Russia, *about fifteen years ago statistics had been almost a target for mockery* (Anuchin 1872, p. 3). O. S.

History of Statistics

56. Survey of the history of science in general

Beginning from the most ancient times we can discern three main periods in the history of science: hierarchical, philosophical and the separation of labour. The last mentioned period can be called the period of systematisation in the full meaning of that word.

The first period covers the time during which sciences remained confined to the temples. Only the priests had been occupied with it. Concealing knowledge from the people, they represented sciences in the guise of emblems.

The second period originated when the sciences, a long time after being transferred from Egypt[1], began to develop in Greece. All at once they had started developing in a completely different direction. They separated themselves from religion and were studied not only by priests but by philosophers as well. These latter informed their contemporaries about the fruits of their investigations, concealed nothing and did not hinder the ensuing delight.

In those times each philosopher covered all the fields of human knowledge. He was at the same time a metaphysicist, a moral admonisher, geometer, naturalist and physicist [and astrologer-astronomer].

The third period was signified by the separation of different branches of science from each other. Each became a special science and the exclusive business of those who wished to devote to it all the power of their mind. Polyhistory ended. Owing to the sensible separation of labour sciences became perfect (?) which was previously impossible even to think about.

That period would have certainly begun earlier had it only depended on Aristotle since that great scholar had set precise and natural boundaries for each science[2]. Regrettably, however, he left no worthy followers[3], whereas in a few centuries the sect of peripatetics which he founded, became contemptible.

And thus the great change in science had not happened until the end of the Middle Ages, in the beginning of the 16th century. Well-considered works and measures directed towards the development of science only date back for three centuries. At the same time statistics, in a systematic form, began to separate itself as a science from political sciences but became independent not before the mid-18th century.

Notes

1. Many more is now known about science in antiquity. I name Neugebauer (1951), and most certainly J. Needham's great monography *Science and Civilization in China* (many volumes and many editions). Then, mathematics in China (Berezkina 1970); in India (Volodarsky 1970); in Babylonia (Berezkina and Youshkevich 1970). Statistics in antiquity had been also studied by many authors, I only mention Hoyrup (2010) and Marrianne (2014). See also Sheynin (2017, Chapter 1). The general sources on the history of mathematics are Cantor (1894 – 1908), and, until the 19th century, Youshkevich (1970 – 1972) and Pearson (1978).

I (1982) discerned three periods in the history of the statistical method. Conclusions were **1)** based on general impression of unregistered observations; **2)** based on registered observations (Graunt, Tycho Brahe); **3)** same, but checked by quantitative criteria. The first period conforms to the qualitative nature of ancient science. Here is an example (Celsus 1935, p. 19):

Careful men noted what generally answered the better and then began to prescribe the same for their patients. Thus sprang up the Art of medicine.
Almost all this also concerns the next sections. O. S.

2. Sciences have common fields with one another. Statistics, for example, cannot be separated from astronomy, meteorology etc. Cf. Note to § 3. O. S.

3. Aristotle had a follower of sorts, Thomas Aquinas who strove to adapt the pagan Philosopher to Christianity. And he attempted to explain the notion of chance and to connect his own theory of probability with the logical and frequentist approaches to it. See Sheynin (1974, pp. 103, 105 and 108) with references to the controversial Byrne (1968) and another reference to a student of Thomas. O. S.

57. Statistics in antiquity

Statistical materials existed from the time when states possessing some enlightenment had originated. For the patriarchal life statistics was certainly not needed at all. Indeed, the people living in a primitive condition constitute a society but not a state. After the people had left their former condition, moved higher and formed states, information about the inner situation of those states had been gradually accumulating. People had been acquainting themselves with the powers at their disposal and applied their observations to the national economy. Egyptians, Jews, Greeks and Romans possessed statistical data about the conditions of their states. Only a proper name was missing. Tables showing the condition of the armies and finance were the first elements of statistics. Then data on the structure and management of the state began to be added.

Greeks and Romans joined this information to politics. During the last periods of the republic and later, under the emperors, statistics for the Romans was the main educational discipline for those young men who devoted themselves to state service and it was then named *notitia publica*.

Gaius Sallustius (*Duae epistolae ad Caesarem*) says [six lines of Latin follow]. Cicero (1928, *On the laws*, III. 8) requires such knowledge from each senator. August and Tiberius [– 42 – 37] wrote such contributions themselves for their own usage as is testified by Sueton (1913, 8.102) and Tacitus (1956, 1.11) respectively.

Everything indicates that Romans had many statistical contributions and teachers of politics as well since young men had been able to learn as seen in Sallustius. Many statistical objects which were called antiquities had been found in the works of ancient authors although only for explaining classical writers, unscrupulously in the political sense and often undated.

58. Statistics in the Middle Ages

During the Middle Ages statistics existed in Rome, Byzantyne, in the Arab world and China. We also find its imprints in the nations which formed states after the Great Migration: in Franks under Carolus (Karolus) Magnus [742? 747? 748? – 814], in the English, under William the Conqueror [1028 – 1087], in the Goths in Spain. However, when the spirit of knighthood spread over Western Europe, arbitrariness destroyed the laws and weakened the states, only then, as it seems, statistics was forgotten.

Nevertheless, by the end of the Middle Ages it originated anew in the Italian republics. Their trade extended over the world as it was then known [not to China!]. Inhabitants of Venice and Genoa took the produce of India, Arabia and the whole Levant and brought it to Europe. They had been in touch with many just consolidated nations. For the sake of the trade they had been compelled to know the economic situation of these nations and collected the pertinent information through their diplomatic agents. At first that information was considered secret and kept in archives, but much became generally known.

Then some had begun to write privately about isolated statistical objects, for example Balducci and Uzano, both from Florence. Silvius (1496) published a book in Germany. Celtes wrote about statistics in prose and verse. Remarkable information about those objects in the Eastern Roman Empire (in the Byzantine Empire) is contained in the works of Byzantine authors. Gibbon collected many appropriate places [passages] and it would have been certainly possible to compile a systematic whole out of them[1].

Note

1. I name two sources on mathematics in those times: Rosenfeld & Youshkevich (1970, pp. 245 – 283; 284 – 326). They are devoted to Europe in the Middle Ages and to the Renaissance respectively. O. S.

59. Statistics of the new time. From Sansovino to Conring

Sansovino (1567) [I established the edition of 1578] was the first really statistical book which described 22 European states. However imperfect it was, it deserved the general approval. Imitations followed, and especially distinguished among them was Botero (1582; 1600). The second book was compiled by many collaborators. D'Avity (1613 or 1616) published a book which had then been considered classical, reprinted many times and translated in other languages. Ranchin in 1635 and Rocoles in 1600 [impossible] provided corrections.

D'Avity was the first in the sequence of French statisticians and France was the first to take over statistics from Venice. Abelin (1616) and De Linda (1663) [I established the edition of 1665] borrowed material from D'Avity. All those contributions were very imperfect and lacked a thorough plan.

60. Statistics of the new time. From Conring to Achenwall

Philology [source criticism] which had been governing in the 16th century was favourable for the mind and prepared the later governing of philosophy. Two new sciences had appeared: natural and civil law.

German politicians, publicists and jurists of the 17th century were quick to note that it was impossible to judge the condition of a state only by reasoning and clearly felt that politics ought to be based on statistical data. Seckendorf (1756) was the first to notice the defects of the current descriptions of states. At the same time, in 1660, the great polyhistor Conring (who died in 1681) announced his lectures at Helmstedt *de rebus publicis nostri aevi celeberrimis* and had thus introduced statistics in the field of university studies.

His contribution was published by Göbel (t. 3 [of Conring's works]). Now it became useless but two other brief considerations (1730a; 1730b) will remain immortal since they contain the embryo of the real theory of statistics. There, Conring was the first to explain how to reveal statistical data [in general descriptions of states] [about three lines in Latin follow]. In all justice, as recognized by Butte and Zizius, Conring has the glory to be called the founder and father of the statistical system since he discovered a criterion of statistical data although was unable to apply it and had not named his science.

His student and follower Oldenburger published his lecture notes (Conring 1675). During this period [until Achenwall] the professors of the new science who lived during and after Conring's lifetime, had published their contributions: Bose, Sagittar, Shubert, Walk in Jena (Walk was Achenwall's contemporary), Kemmerich in Wittenberg [Saxony-Anhalt], Otto in Utrecht and Keler in Altdorf [near Nuremberg] and Göttingen (Бозе, Сагиттар, Вальк, Кеммерих, Келер).

Many books have been published beyond the universities and they show that the notion of statistics had not been established at all. No one was able to show clearly the benefits of all that they taught or wrote and in any case how to apply statistics to a state. Governments paid no attention whatever to these new compilations of historical and statistical information.

61. Continued

At the same time statistical materials speedily accumulated since the inner political life had developed wider and political coups d'état occurred in some states. Such a coup in England in the time of William III [1650 – 1702] in 1689 especially fostered the increase of statistical knowledge about that state. Parliamentary debates, reports of ministries on state revenue and expenses and pertinent studies have explained many government objects which had still been secret in other countries.

From that time onward there appeared many very instructive large and brief contributions. In England, the accumulated public wealth even led to the origin of a special science, political economy. In France, at the same time, the deranged condition of the finances during the second half of the reign of Louis XIV compelled to study deeper after his death the sources of the state revenue and led to the appearance of many contributions in which statistical information was called political (connaisances politiques). In Sweden the same necessity was brought about by the ruinous wars of Charles XII [1682 – 1718] which decreased the population.

The incessantly accumulating statistical materials should have prompted the thinkers to consider their organic unification into a single whole. However, before statistics became an independent science the quantitative political objects gave rise to the origin of political arithmetic[1]. In

England, Graunt, Petty and Davenant [1656 – 1714] had been occupied with it, later it occurred in Holland and France where famous politicians and most celebrated scientists began to study it. Among them were Le Prestre Vauban, De Saint-Pierre [1658 – 1743], Niewentit, Struyck, Kersseboom and 'sGravesande.

The German scientist Süssmilch became especially famous by collecting all their works and discoveries (1741)[2]. Political arithmetic perfected as much as possible [at the time] one of the most important objects of statistics: the cognition of the population. Economists applied political arithmetic to agriculture and English scientists, especially Young, Price and Priestley adapted it to all the branches of national industry.

Notes

1. Yes, Davenant deserved a mention, but much less than Halley whom Obodovsky forgot. Now we believe that Graunt was a statistician (and extremely meritorious he was!). Obodovsky confused statistics and political arithmetic. Confused, for us, was Achenwall (1749, p. 1): he defined the *so-called statistics* as the Staatswissenschaft of separate states, cf. my Introduction. O. S.

2. Obodovsky certainly knew nothing about Süssmilch; his statement was absolutely wrong. O. S.

62. Statistics from Achenwall to our time

In Germany, scientists had been philosophically oriented, in England political enlightenment became widespread. Common and civil law, political economy and political arithmetic became special sciences. And then Achenwall, professor at Göttingen, collected statistical data into a single whole. His contribution (1749) later, in 1752, appeared under another name and ran in three more editions (in 1756, 1762 and 1769) and was posthumously published in 1781 – 1785 and 1790 – 1798 through the efforts of Schlözer and Sprengel.

In all justice, Achenwall initiated a new period in the history of statistics both by that contribution and his university lectures. He was the first to name the new science, to determine better than all his forerunners the notion of statistics and he also partly separated it from geography, metapolitics and history. More than others he hinted at and turned attention to the development of that separation. Achenwall was the first who became able to insert respect to statistics and to extend its study. After him there appeared so many lovers of statistics that a real statistical literature was compiled. It constituted a special branch of studies and filled a few volumes[1].

Note

1. Meusel (*Literatur der Statistik, 1806 und 1807*, Bde 1 – 2) collected all their titles; Niemann, at the end of his theory, showed those most important in a long register. See also Ersch (1813). A. F. Smirdin [1795 – 1857] named Russian statistical contributions in his catalogue of 1806 (with supplements). A. O.

63. Continued

For 90 years after Achenwall statistics has been threatened by various dangers and was not respected everywhere to the same extent. K. F. Hermann (Herrmann) says:

The insufficient political information except that which belonged to the civil law can explain why statistics had been threatened by the opinion that it is a kind of geography and ought to be annihilated. The cause of that opinion was the glorious Geography of Büsching which contained not only geographical, but historical and statistical objects as well.

No one considered that that historical information was a part of history, since it was agreed that geography should be taught together with history. That opinion is even now supported by some French textbooks. However, almost everyone was sure that statistics was only a new name for the previous science, geography. Even in 1804 there appeared in Paris a statistical geography.

Bielefeld and Schlözer had saved statistics. They returned its political direction. The former, in his political instructions, considered statistics as the main part and the foundation of political sciences. The latter had done even more for statistics by creating a special theory for it. He covered all the field of political sciences, duly subdivided it and showed the proper place of statistics. Finally, in his monthly issues (?)*, in correspondence and Political Notices* (Staatsanzeigen)[1] *he practically proved the benefit of statistical information for all political sciences. Thus it was mostly the works of Schlözer which ensured that statistics had avoided the danger of becoming a part of geography*[2].

Another danger threatened statistics. German political calculators dealt thoughtlessly and contrary to the truth with the accumulated numerically expressed material (for example, Krome, Ockhard, Окгард) as well as French and English statisticians and had still more aroused indignation and sneers especially of the Göttingen school (Brandes, Reberg, Germ, Брандес, Реберг, Герм)[3].

That school began to maintain that statistics should not be represented as a soulless skeleton, that it should be supremely directed so that numbers, so important in tables, ought to be banished from it.

They intended to establish a difference between the supreme and the lower statistics and placed the political calculators and linear statisticians in the latter class. But just then political calculators had triumphed: the lower statistics was drawn into state rooms and everywhere in Europe statistical bureaus, offices and even chairs were established. Those calculators did not answer the criticisms of the supreme statisticians whereas the latter quit their attacks. Lüder (1812; 1817) intended to annihilate both the supreme and the lower statistics but only aroused indignation and sneers.

Abuse of numerically expressed statistical objects, statistical calendars and tables harmed statistics[4]. Mechanical minds were especially encouraged and philosophical minds brought to a stop. Statistics became one-sided. There was a time when Europe was flooded, so to say, by statistical tables and calendars so that statistics only consisted of numbers. This circumstance was favourable in that attention was turned on statistical objects and some notion of statistics extended everywhere.

Notes

1. I was unable to establish that source. O. S.

2. Schlözer (1804, § 33) severely criticized Bielefeld but concluded that it would be *an impertinent ingratitude to blame strongly the man who paved the way*.

In § 23bis, Item 8 Schlözer quite favourably commented on Büsching. In § 8 he quoted an author who had noted that many were confusing geography and statistics and stated that, unlike statistics, geography *runs rapidly through one country to another*. In the beginning of § 24bis Schlözer remarked that *we are still not unanimous* […] about the difference between those two sciences. Geography was then understood as an encyclopaedic collection of data on nature, population and economics of various regions. O. S.

3. The grammatical construction of the Russian phrase was wrong and the translation is only probable. O. S.

4. In § 24bis Schlözer justly stated that *general tables were extremely advantageous*. O. S.

64. Continued

Everything done for our science during the long period after Achenwall can be considered under three heads. **1)** Development of the statistical system. **2)** Real statistical studies of states. **3)** Government assistance with the success of statistics.

Due to their extensiveness the first two items constitute a special subject for research (§ 62). Considering the third point it can only be regretted that statistics has belatedly turned the attention of governments to itself. Otherwise it would have reached a higher level of perfection.

At first secretiveness more or less governed in all European offices and most of all hindered success since scientists had been unable to obtain any materials. Statistical researchers were only tolerated but not encouraged. Scientists were allowed to collect all the materials from the published state acts and thus to compile a whole out of fragmentary information which had indeed been the university science, the statistics of scientists, necessarily incomplete and imperfect. More openness reigned only in England due to the conditions of constitutional management[1] and for this reason the political enlightenment had entered Europe from Göttingen (?).

However, from the beginning of this, the 19th century, and indeed from the time of the great Schlözer, governments are turning much more attention to statistics. In many states special statistical offices were established, detailed descriptions of provinces compiled by the order of the governments, land surveyed and censuses carried out, reports and tables issued.

In Russia, in 1805 statistics was included in the educational programmes of gymnasiums and universities. Now, a statistical department is established at the office of the Minister of Interior and statistical committees organized in each province. Yearly reports of the ministries and all the branches of government, periodicals published by the government, the readiness of the offices to provide statistical information, all that furnishes so much statistical materials that we may expect speedy successes and completeness of national statistics in all branches of statistical studies if only the researchers will guide themselves by the true theory.

Note

1. England had and still has no constitution. O. S.

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